CHAPTER

7

Trigonometry of Right Triangles

Consider the riders on the Drop Zone ride. How high in the air do you think they are? Did you know that you could calculate this height by examining the shadows cast on the ground? All you need is a metre stick and a little understanding of the geometry of triangles.

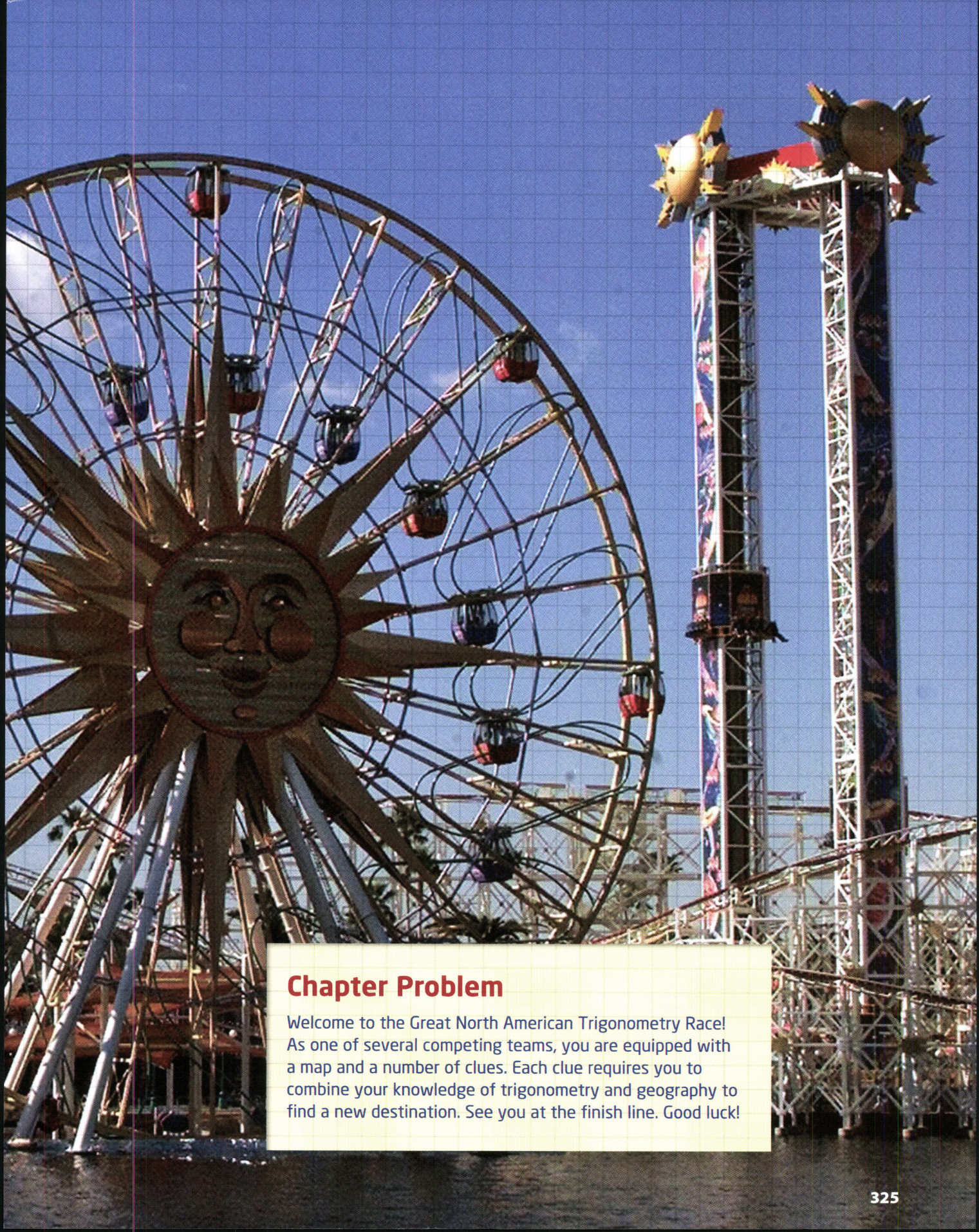
In ancient times, astronomers studied the motion of the stars and planets and wondered about how far away they were and how fast they were travelling. Eventually, techniques were discovered that allowed them to determine such measures right here on Earth! The branch of mathematics they invented to do this is called trigonometry.

Trigonometry

- Verify, through investigation, the properties of similar triangles.
- Describe and compare the concepts of similarity and congruence.
- Solve problems involving similar triangles in realistic situations.
- Determine, through investigation, the relationship between the ratio of two sides in a right triangle and the ratio of the two corresponding sides in a similar right triangle, and define the sine, cosine, and tangent ratios.
- Determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios and the Pythagorean theorem.
- Solve problems involving the measures of sides and angles in right triangles in real-life applications, using the primary trigonometric ratios and the Pythagorean theorem.

Vocabulary

congruent figures
similar figures
isometry
scale factor, k
slope angle
tangent of an angle
primary trigonometric ratios
angle of depression
angle of elevation



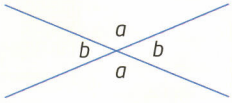
Chapter Problem

Welcome to the Great North American Trigonometry Race! As one of several competing teams, you are equipped with a map and a number of clues. Each clue requires you to combine your knowledge of trigonometry and geography to find a new destination. See you at the finish line. Good luck!

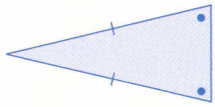
Get Ready

Angle Properties

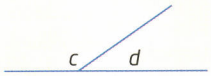
When two lines intersect, the **opposite angles** are equal.



The angles opposite the equal sides of an isosceles triangle are equal.

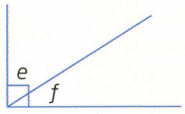


Supplementary angles sum to 180°.



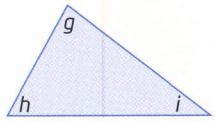
$c + d = 180°$

Complementary angles sum to 90°.



$e + f = 90°$

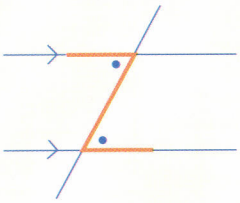
The sum of the interior angles in a triangle is 180°.



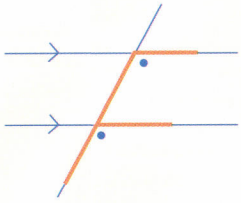
$g + h + i = 180°$

When a transversal crosses parallel lines, many pairs of angles are related.

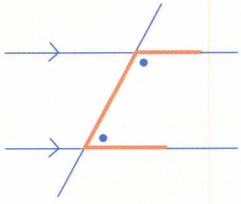
alternate angles are equal



corresponding angles are equal

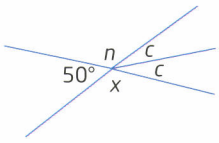


co-interior angles are supplementary



1. Find each unknown angle.

a)



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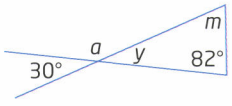

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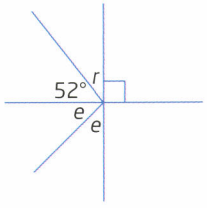
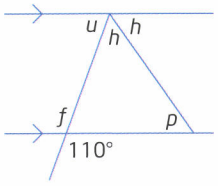
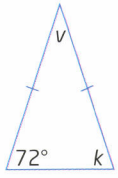


d)

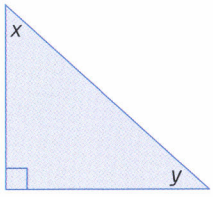


e)

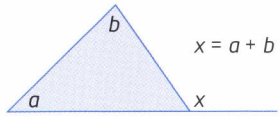


2. Prove that each interior angle of an equilateral triangle measures 60°.

3. Prove that for any right triangle, the two acute angles are complementary.



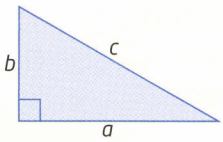
4. The exterior angle theorem states that the exterior angle of a triangle is equal to the sum of the two opposite interior angles.



Prove the exterior angle theorem.

Pythagorean Theorem

The three side lengths of a right triangle are related by the equation $c^2 = a^2 + b^2$.



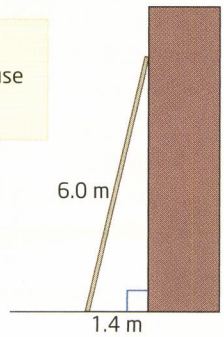
How far up the wall will the ladder reach?

Substitute the known values into the Pythagorean theorem.

$$c^2 = a^2 + b^2$$
$$6.0^2 = 1.4^2 + b^2$$
$$36 = 1.96 + b^2$$
$$36 - 1.96 = b^2$$
$$34.04 = b^2$$
$$\sqrt{34.04} = \sqrt{b^2}$$
$$5.83 \doteq b$$

I have to substitute carefully in the Pythagorean theorem. The square of the length of the hypotenuse usually appears alone on one side of the equation.

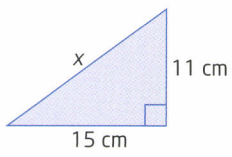
Subtract 1.96 from both sides of the equation.



The ladder will reach approximately 5.8 m up the wall.

5. Find the unknown side length in each triangle. Round your answers to the nearest tenth of a unit.

a)



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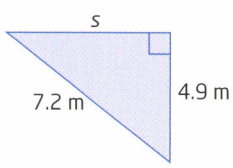
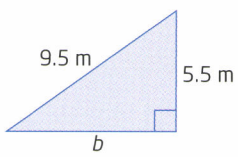


6. Find the unknown side length in each triangle. Round your answers to the nearest tenth of a unit.

a)



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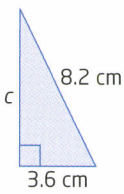
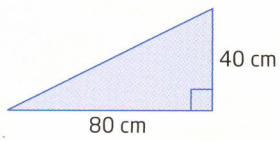


Slope

The slope of a line or line segment is the ratio of its rise to its run. The letter m is often used to indicate slope.

The slope of the ramp can be found by calculating this ratio.

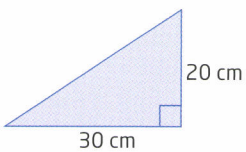
$$m = \frac{\text{rise}}{\text{run}}$$
$$= \frac{40}{80}$$
$$= \frac{1}{2} \text{ or } 0.5$$



Slope can be expressed as a fraction in lowest terms or as a decimal.

7. Find the slope of each ramp.

a)



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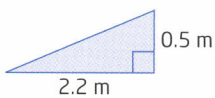
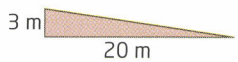


8. For safety reasons, a ramp should have a slope of not more than $\frac{1}{12}$. Determine whether each ramp is safe.

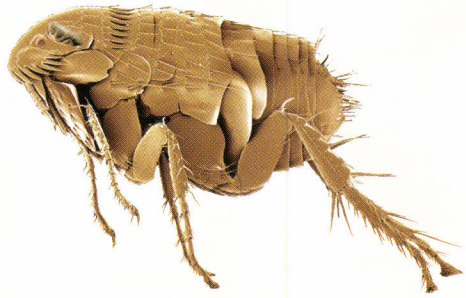
a)



b)

Equivalent Ratios

A ratio compares two quantities with the same units. For example, the photograph of a flea has been magnified 30 times.



The ratio 1:30 describes the relationship between the actual size and the image size of the flea. You can write this as a fraction:

$\frac{1}{30}$ ← actual size
$\frac{1}{30}$ ← image size

The image length of the flea is 45 mm. Using equivalent ratios, you can find the actual length, L, of the flea.

$$\frac{\text{actual}}{\text{image}} = \frac{1}{30}$$

$$\frac{L}{45} = \frac{1}{30}$$

$$L = 45\left(\frac{1}{30}\right) \quad \text{Multiply both sides by 45.}$$

$$L = 1.5$$

The actual length of the flea is 1.5 mm.

9. Solve each proportion.

a) $\frac{x}{6} = \frac{1}{2}$ **b)** $\frac{3}{y} = \frac{2}{3}$

c) $\frac{a}{2} = \frac{2}{a}$ **d)** $\frac{x+2}{2} = \frac{4}{x}$

10. Pick two parts of your choice on the flea. Measure their image lengths. Use equivalent ratios to find their actual lengths.

11. **Chapter Problem** Before looking at your first clue and beginning the race, you need to understand how to read distances on your map. Look at the scale in the bottom right corner.

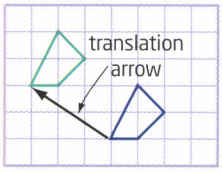
a) Write a ratio to describe the relationship between map distance and actual distance.

b) Measure the map distances for the following. Then, find the actual distances, to the nearest 100 km.
- Moosonee to Ottawa
- Toronto to Miami

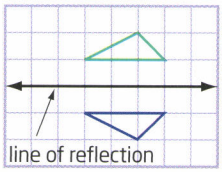
c) Pick two other points on the map. Determine how far apart they are.

Transformations

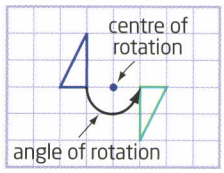
The diagrams illustrate the images produced when a shape is transformed in different ways.



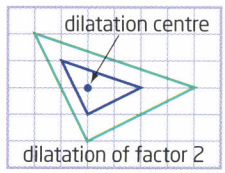
A **translation** is a slide along a fixed distance and direction.



A **reflection** is a flip across a mirror line, or line of reflection.



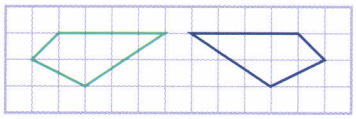
A **rotation** is a turn about a centre point, or centre of rotation.



A **dilatation** is an enlargement or reduction by a fixed factor, measured from a centre point.

12. Identify the type of transformation in each case.

a)



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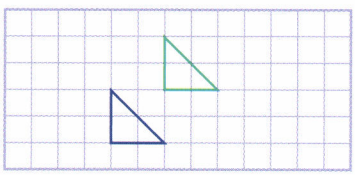
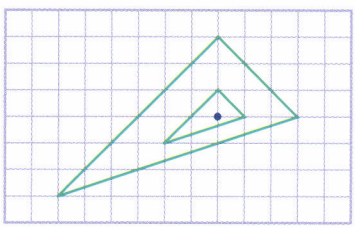


c)



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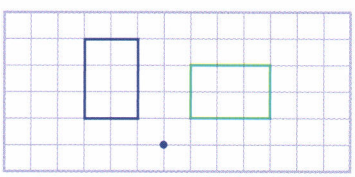


13. Draw a right triangle and label its vertices. Draw a triangle that is

a) a rotated image of the original

b) a dilated image of the original

c) a reflected image of the original

d) a translated image of the original

14. Draw an acute triangle and label its vertices. Draw a triangle that is

a) a rotated image of the original

b) a dilated image of the original

c) a reflected image of the original

d) a translated image of the original

7.1 Investigate Properties of Similar Triangles

Geometric shapes are often used in construction and design. Not only are certain shapes naturally pleasing to the eye, but they are also often useful for their structural properties.

Consider the shed shown. What geometric shapes can you recognize? Identify at least one pair of

- **congruent figures**
- **similar figures**

congruent figures
- identical shapes
- same size

similar figures
- identical shapes
- different sizes

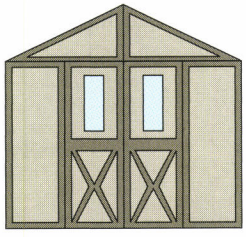
Tools
- copy of shed drawing
- ruler
- protractor

Optional
- tracing paper

Investigate

How can I recognize similar and congruent figures?

Look at the front face of the shed.



1. **a)** What pairs of congruent figures can you find?

   b) Copy two or three pairs of congruent figures from the drawing of the shed. Label the vertices of each figure. What special properties do the corresponding sides and angles have?

2. **Reflect** Summarize the properties of congruent triangles.

3. **a)** What pairs of similar, but not congruent, figures can you find?

   b) Make copies of two or three pairs of similar figures from the drawing of the shed. Label the vertices of each figure. What special properties do the corresponding sides and angles have?

4. **Reflect**

   a) What is true about the corresponding angles in two congruent figures? in two similar figures?

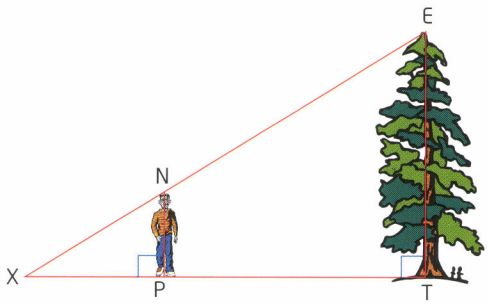
   b) What is true about the corresponding side lengths in two congruent figures? in two similar figures?

   c) Copy and complete the table to summarize your answers to parts a) and b).

	Corresponding Angles Are ...	Corresponding Side Lengths Are ...
Congruent Figures		
Similar Figures		

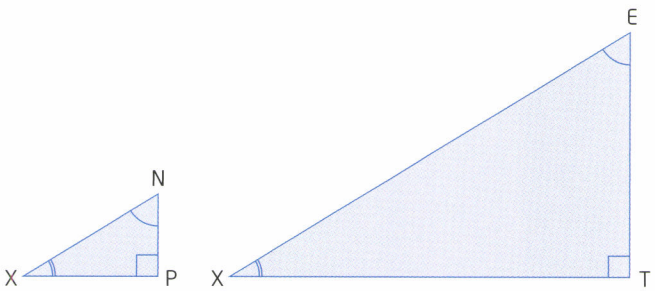
Example 1 Use Angles to Show That Two Triangles Are Similar

Identify a pair of similar triangles and explain why they are similar. Support each statement with a reason.



Solution

There are two overlapping similar triangles: $\triangle XPN$ and $\triangle XTE$. The diagram shows them separated:



You can show that these two triangles are similar by identifying three pairs of equal angles.

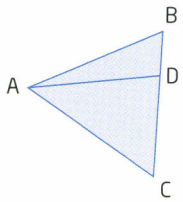
You can see three pairs of equal angles.

Statement	Reason
$\angle XPN = \angle XTE$	Both are 90°.
$\angle PXN = \angle TXE$	These are the same angle.
$\angle XNP = \angle XET$	The sum of the angles in any triangle is 180°.
$\triangle XPN \sim \triangle XTE$	All three pairs of angles are equal.

The third pair of angles in any two triangles is equal if the other two pairs are equal. So, it is only necessary to have two pairs of equal corresponding angles to show that two triangles are similar.

Literacy Connections

In $\triangle ABC$, the interior angle at vertex B can be called $\angle ABC$, $\angle CBA$, or $\angle B$. In the first two cases, the middle letter corresponds to the vertex of the angle.



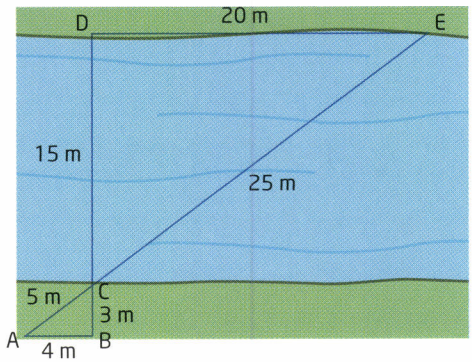
The three-letter system is useful to describe the three interior angles at vertex A: $\angle BAD$, $\angle DAC$, and $\angle BAC$. Referring simply to $\angle A$ can lead to confusion.

Literacy Connections

The symbol ~ means "is similar to." When you write a similarity statement, the order of the vertices must correctly identify pairs of equal angles and pairs of corresponding sides. In Example 1, $\triangle XPN \sim \triangle XTE$ is correct because the vertices correctly indicate pairs of equal angles and pairs of corresponding sides.

Example 2 Use Sides to Show That Two Triangles Are Similar

Are the two triangles in the diagram similar? Explain your reasoning.



Solution

You can show that two triangles are similar by showing that three pairs of corresponding sides are proportional.

The two triangles are $\triangle EDC$ and $\triangle ABC$.

$$\frac{ED}{AB} = \frac{20}{4} \qquad \frac{CE}{CA} = \frac{25}{5} \qquad \frac{DC}{BC} = \frac{15}{3}$$

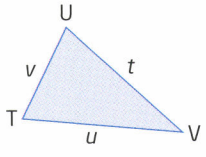
$$= \frac{5}{1} \qquad = \frac{5}{1} \qquad = \frac{5}{1}$$

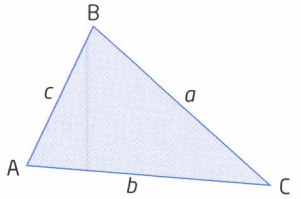
The three pairs of corresponding sides are all in the ratio 5:1. Therefore, $\triangle EDC \sim \triangle ABC$.

Since the triangles in Example 2 are similar, their corresponding angles are equal. That is, $\angle CAB = \angle CED$, $\angle CBA = \angle CDE$, and $\angle ACB = \angle ECD$.

Key Concepts

- Congruent figures have the same size and shape. Similar figures have the same shape, but different sizes.
- Similar triangles have the following properties:
  - Corresponding angles are equal:
    $\angle A = \angle T$
    $\angle B = \angle U$
    $\angle C = \angle V$
  - Ratios of corresponding sides are equal:
    $\frac{AB}{TU} = \frac{BC}{UV} = \frac{CA}{VT}$, or, in ratio notation, AB:TU = BC:UV = CA:VT





Communicate Your Understanding

C1 **a)** Explain the difference between similar figures and congruent figures.

b) Draw an example for each.

C2 **a)** How are the angles of similar figures related?

b) How are the side lengths of similar figures related?

c) How do these relationships differ from those involving congruent figures?

C3 Describe two ways that you can show that two triangles are similar.

C4 Why is it only necessary to show that two pairs of corresponding angles are equal in order to show that two triangles are similar?

Practise

For help with questions 1 to 6, see the Investigate.

1. Look in the room around you. Identify at least two sets of

a) congruent figures

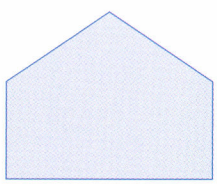
b) similar figures

2. Identify at least two examples outside the classroom of where you see

a) congruent figures

b) similar figures

3. **a)** Copy the diagram.



b) Draw each of the following:

- a figure congruent to the one in part a)
- a smaller figure that is similar to the one in part a)
- a larger figure that is similar to the one in part a)

c) Verify the accuracy of your sketches by measuring angles and side lengths.

4. For each situation, identify whether you would use similar figures, congruent figures, or neither. Justify your answers.

a) a tile pattern on a floor

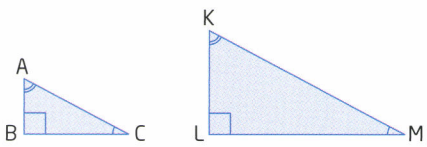
b) a team's logo for the chest and shoulder patches of their jerseys

c) a door frame and a window frame for the front of a house

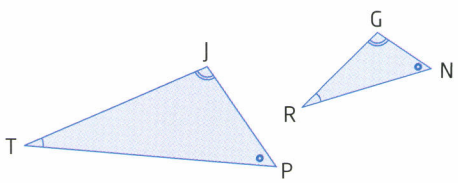
d) a three-dimensional model of a skyscraper building

5. Name the similar triangles in each case. Write the letters so that equal angles appear in corresponding order.

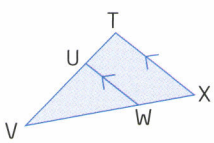
a)



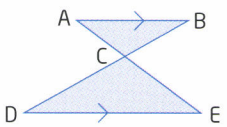
b)



c)



d)

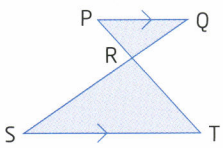


6. For each pair of similar triangles in question 5, write the equivalent ratios of side lengths.

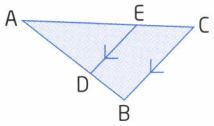
For help with questions 7 to 9, see Examples 1 and 2.

7. Name a pair of similar triangles in each diagram and explain why they are similar.

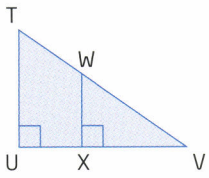
a)



b)

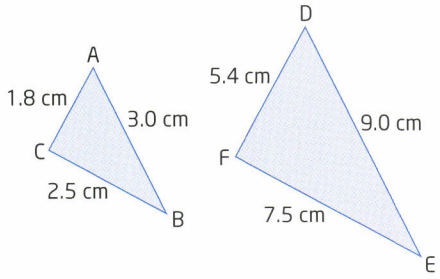


c)

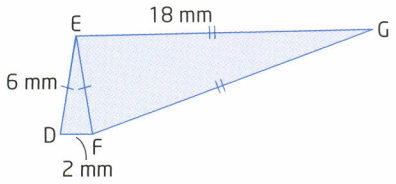


8. Name a pair of similar triangles in each diagram and explain why they are similar.

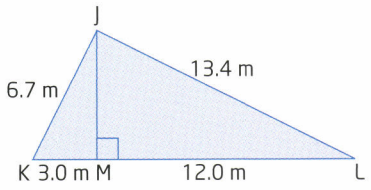
a)



b)



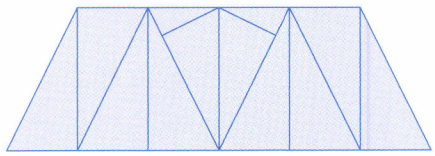
c)



9. For each pair of similar triangles in questions 7 and 8, list all the pairs of corresponding angles and corresponding sides.

Connect and Apply

Carefully copy or trace the diagram of the truss bridge. Use it to answer questions 10 to 12.



10. Identify an example of each of the following. Use different colours and/or additional sketches to illustrate your answers.

a) a pair of congruent triangles

b) a pair of congruent rectangles

c) a pair of congruent parallelograms

d) a pair of similar triangles

e) a set of three similar triangles

11. Identify a pair of congruent triangles that are related by a

a) translation

b) reflection

c) rotation

12. a) Describe the combination of transformations that relates one pair of congruent triangles.

b) Describe a different combination of transformations that relates another pair of congruent triangles.

13. a) Draw a right triangle.

b) Draw a right triangle that is

- congruent to the one you drew
- similar to the one you drew
- neither congruent nor similar to the one you drew

14. Are all equilateral triangles similar? Justify your answer.

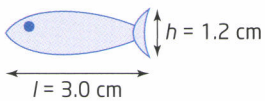
15. Are all isosceles triangles similar? Justify your answer.

Extend

16. Congruent figures occur in music composition. This is a two-bar lead-in that was commonly used in the early days of rock 'n' roll music:

Carefully copy the music into your notebook. Then, use line segments to join the centres of the note heads (the round part) so they form two congruent triangles. What kind of transformation relates the two triangles?

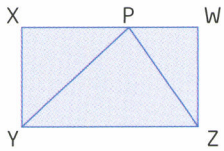
17. A type of bass grows so that it retains its shape. Its length, height, and width all remain in the same proportion for its first few years of life. The average length, l, and height, h, of a baby bass are shown. The average width, w, of this type of bass is about half its height. If the bass triples in length each month for the first 3 months of life, find the dimensions of this bass after



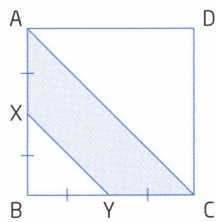
a) 1 month **b)** 2 months

18. Refer to question 17. The density of this type of bass is about 10% greater than that of water, which is 1 g/cm^3. Use the relationship density $= \frac{\text{mass}}{\text{volume}}$ to estimate the mass of this bass after 3 months. To estimate the volume of the bass, find the volume of a rectangular prism with the same width, length, and height as the bass.

19. Math Contest XYZW is a rectangle, with YZ = 10 cm, YP = 8 cm, and ZP = 6 cm. Determine the area of XYZW.



20. Math Contest ABCD is a square. X is the midpoint of AB and Y is the midpoint of BC. What percent of the square is shaded?



A 37.5% **B** 25%

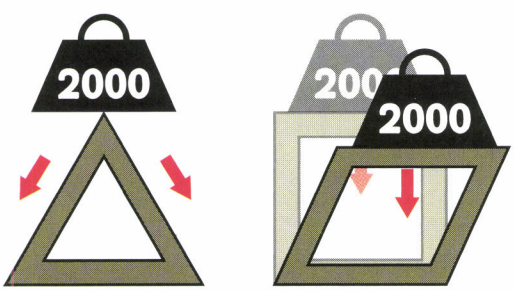
C 18.75% **D** $33\frac{1}{3}\%$ **E** 30.75%

Making Connections

The triangle is the strongest polygon structurally. The only way you can change its shape is to change the length of one or more of its sides. A square, by contrast, can be deformed into a rhombus under a heavy load.

It is this property that makes the triangle so popular in designs such as truss bridges.

The Quebec Bridge, shown here, which spans the St. Lawrence River, is the longest cantilever truss bridge in the world.



Go to www.mcgrawhill.ca/links/principles10 and follow the links to learn more about the Quebec Bridge and its fascinating history.

Create Designs With Similar and Congruent Figures Using Dynamic Geometry Software

Investigate

How can you use dynamic geometry software to create and use similar and congruent figures?

A: Create and Transform Congruent and Similar Figures

Tools

- computer with *The Geometer's Sketchpad®*

Method 1: Use *The Geometer's Sketchpad®*

1. a) Open *The Geometer's Sketchpad®* and begin a new sketch.

b) Construct a triangle.
- Plot three points.
- Select all of the points.
- From the **Construct** menu, choose **Segments**. A triangle will appear.
- Select the points. From the **Construct** menu, choose **Triangle Interior**. You can change the colour by right clicking on the interior that appears and choosing a colour.

c) Measure the side lengths, all at once.
- Select the three sides.
- From the **Measure** menu, choose **Length**.

d) Measure the angles, one at a time.
- Select three vertices, taking careful note of the order. The middle vertex you select is where the angle will be measured.
- From the **Measure** menu, choose **Angle**.
- Repeat for the other two angles.

e) Measure the area.
- Select the triangle interior.
- From the **Measure** menu, choose **Area**.

You can click and drag the measures to place them wherever you like.

f) Click and drag one or more of the vertex points. Notice that the measures change dynamically as you change the triangle.

2. Return to page 1 of your sketch by clicking on the **1** at the bottom left of the sketch. Create an image triangle using a translation.

a) Select all parts of the triangle, including the
- vertices
- line segments
- interior

Technology Tip

To avoid having to create a new triangle for each part of this activity, attach three more pages to your sketch with the same triangle on each page.
- From the **File** menu, choose **Document Options**.
- Click on **Add Page**, choose **Duplicate**, and then choose **1**.
- Repeat these steps to add two more pages.

b) From the **Transform** menu, choose **Translate**. A dialogue box will appear.

c) Experiment with different translations. Explain the difference between **Polar** and **Rectangular** translations.

d) Is the resulting triangle similar to the original, congruent to the original, or neither? Verify by measuring.

e) What happens to the area of the triangle after the translation? Explain.

f) Click and drag one or more of the vertices of either triangle and describe what happens.

3. Go to page 2 of your sketch. Create an image triangle using a reflection.

a) Draw a line segment near the triangle.

b) From the **Transform** menu, choose **Mark Mirror**.

c) Select all parts of the triangle. From the **Transform** menu, choose **Reflect**. Explain what happens.

d) Is the resulting triangle similar to the original, congruent to the original, or neither? Verify by measuring.

e) What happens to the area of the triangle after the reflection? Explain.

f) Click and drag one or more vertices of either triangle and describe what happens.

g) Click and drag either vertex of the mirror line and describe what happens.

4. Go to page 3 of your sketch. Create an image triangle using a rotation.

a) Construct a point near the triangle.

b) From the **Transform** menu, choose **Mark Center**.

c) Select all parts of the triangle. From the **Transform** menu, choose **Rotate**. A dialogue box will appear. Enter an angle of your choice. Click on **Rotate** and explain what happens.

d) Is the resulting triangle similar to the original, congruent to the original, or neither? Verify using measurements.

e) What happens to the area of the triangle after the rotation? Explain.

f) Click and drag one or more vertices of either triangle and describe what happens.

g) Click and drag the centre of rotation and describe what happens.

h) Create another image triangle by rotating in the opposite direction by changing the sign of the angle when the **Rotate** dialogue box appears.

Technology Tip

To select multiple objects that are close together, follow these steps.

- Place the cursor above and to one side of all the objects you want to select.
- Click and drag to a point below and to the other side of the objects. A dashed box will appear.
- Release the mouse button. Any objects within the box will be selected.

5. Go to page 4 of your sketch. Create an image triangle using a dilatation.

a) Construct a point near the triangle.

b) From the **Transform** menu, choose **Mark Center**.

c) Select all parts of the triangle. From the **Transform** menu, choose **Dilate**. A dialogue box will appear. Dilate by a fixed ratio of your choice by entering values into the numerator and denominator fields. Click **Dilate** and explain what happens.

d) Is the resulting triangle similar to the original, congruent to the original, or neither? Verify using measurements.

e) What happens to the area of the triangle after the dilatation? How is the new area related to the ratio you chose in part c)?

f) Click and drag one or more vertices of either triangle and describe what happens.

g) Create new triangles using dilatations. Experiment with different values in the numerator and denominator of the **Dilate** dialogue box. Describe what happens when

- numerator < denominator
- numerator > denominator
- numerator = denominator

How is this ratio related to the ratio of corresponding side lengths of the original and image triangles?

Literacy Connections

The Geometer's Sketchpad® and Cabri® Jr. use the term "dilation" instead of "dilatation." The two words mean the same thing.

Tools

- TI-83 Plus or TI-84 Plus graphing calculator

Method 2: Use a Graphing Calculator

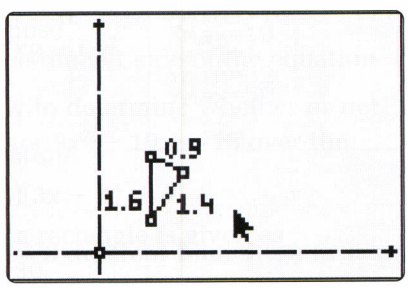
1. a) Press APPS and choose **Cabri Jr**. If the axes are not visible, choose the **F5** menu. Choose **Hide/Show**, and then **Axes**. Move the cursor to the origin. Press ALPHA, and move the origin until the first quadrant occupies most of the screen, as shown.

b) Construct a triangle.

- Choose **Triangle** from the **F2** menu. Draw a triangle as shown.

c) Measure the side lengths of the triangle.

- Choose **Measure** from the **F5** menu. Then, choose **D. & Length**.
- Move the cursor to one of the sides of the triangle until it flashes. Press ENTER. Move the measurement to a suitable location, and press ENTER again.
- Repeat this procedure to measure the lengths of the other two sides.



d) Measure the angles, one at a time.

- Choose **Measure** from the **F5** menu. Then, choose **Angle**.
- Move the cursor to one of the vertices of the triangle. Press ENTER. Move to the next vertex. Press ENTER. This is the middle vertex for the angle being measured. Move to the third vertex. Press ENTER. Move the measurement to a convenient location, and press ENTER.
- Repeat this procedure to measure the two remaining angles.

e) Measure the area.

- Choose **Measure** from the **F5** menu. Then, choose **Area**.
- Move the cursor to one of the sides of the triangle until it flashes. Press ENTER. Move the area measurement to a convenient location, and press ENTER.

f) Move the cursor to one of the vertices of the triangle. Press ALPHA. Drag the vertex. Note how the measurements change.

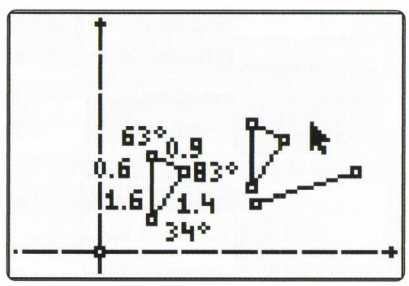
g) Save your sketch.

- Choose **Save As ...** from the **F1** menu. Enter a suitable name. Press ENTER.

2. Create an image triangle using a translation.

a) Choose **Segment** from the **F2** menu. Draw a line segment, as shown. This segment indicates the direction and length of the translation.

b) From the **F4** menu, choose **Translation**. Move the cursor to one of the sides of the triangle until it flashes. Press ENTER. Move the cursor to the line segment that defines the translation. Press ENTER. The translated triangle will appear.



c) Is the resulting triangle similar to the original, congruent to the original, or neither? Verify using measurements.

d) What happens to the area of the triangle after the translation? Explain.

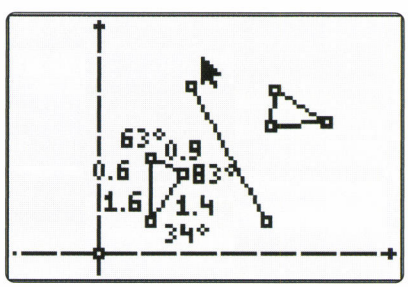
e) Select and drag one or more of the vertex points of the original triangle and describe what happens.

3. Create an image triangle using a reflection.

a) Open the file that you saved in step 1 g). When you are asked whether you want to save the changes, choose **No**.

b) Choose **Segment** from the **F2** menu. Draw a line segment, as shown. This segment serves as the mirror line.

c) From the **F4** menu, choose **Reflection**. Move the cursor to one of the sides of the triangle until it flashes. Press ENTER. Move the cursor to the mirror line. Press ENTER. The reflected triangle will appear.



d) Is the resulting triangle similar to the original, congruent to the original, or neither? Verify using measurements.

e) What happens to the area of the triangle after the reflection? Explain.

f) Select and drag one or more vertices of the original triangle and describe what happens.

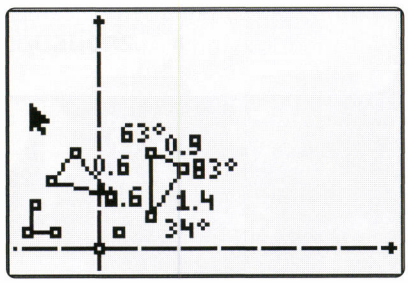
g) Select and drag either endpoint of the mirror line and describe what happens.

4. Create an image triangle using a rotation.

a) Open the file that you saved in step 1 g). When you are asked whether you want to save the changes, choose **No**.

b) Choose the **Point** tool from the **F2** menu. Draw a point near the origin to serve as the centre of rotation. Draw three points to define the angle of rotation.

c) From the **F4** menu, choose **Rotation**. Move the cursor to one of the sides of the triangle until it flashes. Press ENTER. Move the cursor to the point that defines the centre of rotation. Press ENTER. Select the three points that define the angle of rotation. Press ENTER. The rotated triangle will appear.



d) Is the resulting triangle similar to the original, congruent to the original, or neither? Verify using measurements.

e) What happens to the area of the triangle after the rotation? Explain.

f) Select and drag one or more vertices of the original triangle and describe what happens.

g) Select and drag the centre of rotation and describe what happens.

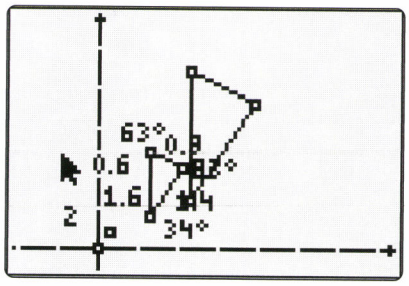
h) Select and drag one of the ends of the angle of rotation, and describe what happens.

5. Create an image triangle using a dilatation.

a) Open the file that you saved in step 1 g). When you are asked whether you want to save the changes, choose **No**.

b) Choose the **Point** tool from the **F2** menu. Draw a point near the origin to serve as the centre of dilatation. Choose the **Alpha-Num** tool from the **F5** menu. Move to a convenient location to place the dilatation factor. Press ENTER and then ALPHA. Use a dilatation factor of 3. Press ENTER.

c) From the **F4** menu, choose **Dilation**. Move the cursor to one of the sides of the triangle until it flashes. Press ENTER. Move the cursor to the point that defines the centre of dilatation. Press ENTER. Move the cursor to the dilatation factor. Press ENTER. The dilated triangle will appear.



d) Is the resulting triangle similar to the original, congruent to the original, or neither? Verify using measurements.

e) What happens to the area of the triangle after the dilatation? How is the new area related to the dilatation factor in part b)?

f) Select and drag one or more vertices of the original triangle and describe what happens.

g) Select and drag the centre of dilatation and describe what happens.

h) Start with a new triangle, and use a different dilatation factor. Try a factor between 0 and 1.

B: Create a Design

1. Choose an object that you would like to design, such as a building, a bridge, or a vehicle.

2. Create your design, using the following guidelines.
   - Include at least two sets of congruent shapes.
   - Include at least two sets of similar shapes.
   - Use at least three different types of transformations.

   a) Find the ratio that relates corresponding sides for each pair of similar shapes.

   b) Find the ratio of areas for each pair of similar shapes, and identify how this value in each case is related to the ratio in part a).

3. **Reflect**

   a) Describe the processes you used to create similar and congruent objects in your design.

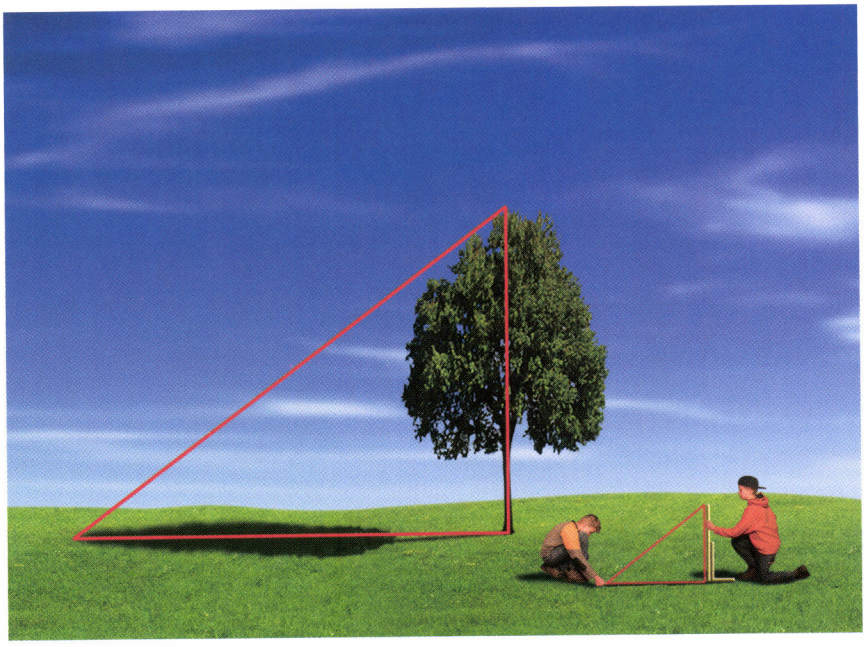
   b) Explain how you can prove that two objects are either similar or congruent.

   c) Which types of transformations are **isometries**? Which are not? Explain your reasoning.

isometry

- a transformation that preserves size and shape

7.2 Use Similar Triangles to Solve Problems



The geometry of similar figures is a powerful area of mathematics. Similar triangles can be used to measure the heights of objects that are difficult to get to, such as trees, tall buildings, and cliffs. They can also be used to measure distances across rivers and even galaxies!

The students in the photo are using a metre stick and shadows to measure the height of the tree. How can they do this? What role do similar triangles play in this type of problem?

Investigate

Tools

- ruler
- metre stick

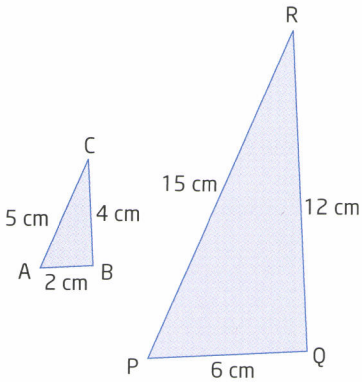
How can you apply the properties of similar triangles to solve problems?

Suppose you have a metre stick and it is sunny outside. Your task is to plan a problem solving strategy so that you can determine the height of an inaccessible object, such as a tree, a building, or a cliff.

1. Look at the illustration. Discuss with a partner, or in a small group, how the students could find the height of the tree.
2. Draw a diagram that relates to this problem. Explain how similar triangles are involved.
3. Create some numbers to represent reasonable measures to solve this problem and solve it. Does your answer seem reasonable? Explain.
4. **Reflect** Trade strategies with another pair or group. Compare strategies. Do you think they will work? Make any improvements you like to your own strategy. Later you will apply your method to solve a real measurement problem.

The **scale factor, k**, is a useful quantity when working with similar triangles such as the ones shown.

The value of k relating corresponding sides in these two triangles is 3, because if you multiply each side length in $\triangle ABC$ by 3, you obtain the corresponding side length in $\triangle PQR$.



scale factor, k

- factor that relates corresponding side lengths of two similar triangles

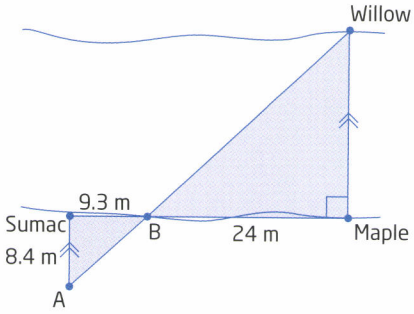
$\triangle ABC$ Side Lengths (cm)	Multiply by k	$\triangle PQR$ Side Lengths (cm)
AB = 2	$2 \times 3 = 6$	PQ = 6
BC = 4	$4 \times 3 = 12$	QR = 12
CA = 5	$5 \times 3 = 15$	RP = 15

You can apply the scale factor to find an unknown side length in one triangle if you know the corresponding side length in a similar triangle.

Example 1 Solve for an Unknown Side

To determine the width of a river, Naomi finds a willow tree and a maple tree that are directly across from each other on opposite shores. Using a third tree on the shoreline, Naomi plants two stakes, A and B, and measures the distances shown.

Find the width of the river using the information that Naomi found.

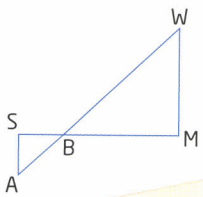


Solution

You can find two similar triangles and then use the scale factor to find the width.

Step 1: Show that $\triangle ABS$ is similar to $\triangle WBM$.

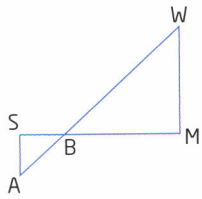
Statement	Reason
$\angle ABS = \angle WBM$	These are opposite angles.
$\angle BSA = \angle BMW$	These are both 90°, because AS is parallel to WM.
$\triangle ABS \sim \triangle WBM$	Two pairs of corresponding angles are equal.



If two pairs of corresponding angles are equal, I know that the angles in the third pair are also equal.

Step 2: Find the scale factor.

In $\triangle ABS$ and $\triangle WBM$, BS and BM are corresponding sides. Their ratio gives the scale factor, k:

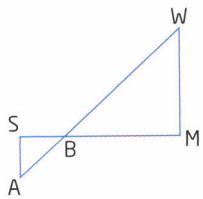


$$k = \frac{BM}{BS}$$
$$= \frac{24}{9.3}$$

Each side in the larger triangle is $\frac{24}{9.3}$ times as long as its corresponding side in the smaller triangle. Leave the scale factor in this form for more accuracy in later calculations.

Step 3: Find the width of the river.

In $\triangle ABS$ and $\triangle WBM$, AS and WM are corresponding sides. Use the scale factor to find WM.



$$\frac{WM}{AS} = k$$
$$\frac{WM}{8.4} = \frac{24}{9.3}$$
$$WM = \frac{24}{9.3}(8.4) \quad \text{Multiply both sides by 8.4.}$$
$$\doteq 21.667$$

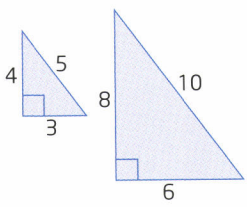
21.677 m is too precise an answer based on the measurements given. One of the measures, BM, is only accurate to the nearest metre. So, it is reasonable to round the answer to the nearest metre.

Therefore, the width of the river is approximately 22 m.

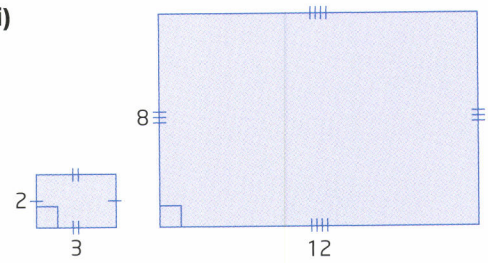
Example 2 Areas of Similar Figures

a) What is the relationship between the areas in each pair of similar figures?

i)



ii)



b) Find the scale factor, k, for each pair of figures.

c) Compare your answers to parts a) and b).

Solution

a) Find the areas of the figures.

i) Smaller Triangle

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(3)(4)$$
$$= 6$$

The area of the smaller triangle is 6 square units.

Larger Triangle

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(6)(8)$$
$$= 24$$

The area of the larger triangle is 24 square units.

The area of the larger triangle is 4 times the area of the smaller triangle.

ii) Smaller Rectangle

$$A = l \times w$$
$$= 2 \times 3$$
$$= 6$$

The area of the smaller rectangle is 6 square units.

Larger Rectangle

$$A = l \times w$$
$$= 8 \times 12$$
$$= 96$$

The area of the larger rectangle is 96 square units.

The area of the larger rectangle is 16 times the area of the smaller rectangle.

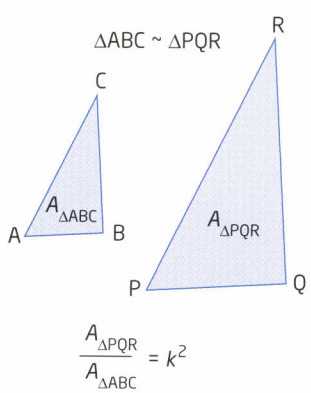
b) i) Since each side length of the larger triangle is 2 times the length of the corresponding side of the smaller triangle, the scale factor is $k = 2$.

ii) Since each side length of the larger rectangle is 4 times the length of the corresponding side of the smaller rectangle, the scale factor is $k = 4$.

c) In both cases, the ratio of the area of the larger figure to the area of the smaller figure is equal to the square of the scale factor, k.

This relationship holds for all similar figures: the ratio of the areas of two similar figures is equal to the square of the scale factor.

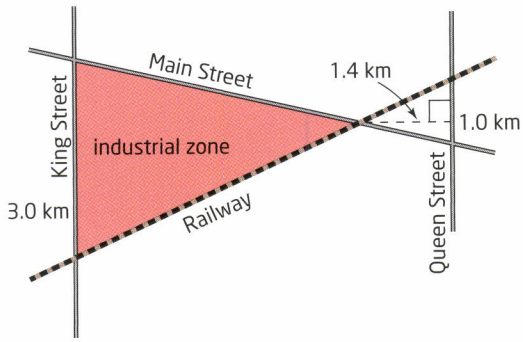
Another way to write this is $A_{\triangle PQR} = k^2(A_{\triangle ABC})$. You can use this to solve for an unknown area.



Example 3 Solve for an Unknown Area

The shaded area is to be an industrial zone.

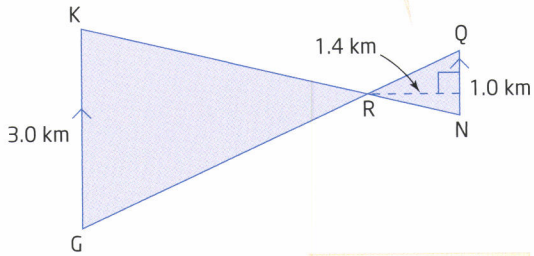
Find the area of the industrial zone. Assume that King and Queen are parallel and that all streets and the track are straight.



Solution

Identify two similar triangles. Find the scale factor and use it to find the area of the larger triangle.

I'll make a simplified diagram.



Statement	Reason
$\angle KRG = \angle NRQ$	Opposite angles are equal.
$\angle RKG = \angle RNQ$	Alternate angles are equal.
$\triangle KRG \sim \triangle NRQ$	Corresponding angles are equal.

If two pairs of corresponding angles are equal, I know that the angles in the third pair are equal, too.

The scale factor is equal to the ratio of corresponding sides:

$$k = \frac{KG}{NQ}$$
$$= \frac{3.0}{1.0}$$
$$= 3$$

Find the area of the smaller triangle using the given information.

$$A_{\triangle NRQ} = \frac{1}{2}bh$$

Apply the formula for the area of a triangle.

$$= \frac{1}{2}(1.0)(1.4)$$
$$= 0.7$$

The area of the smaller triangle is 0.7 km^2. Use this and the scale factor, $k = 3$, to find the area of the larger similar triangle.

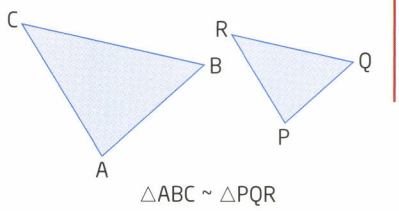
$$A_{\triangle KRG} = k^2(A_{\triangle NRQ})$$
$$= 3^2(0.7)$$
$$= 9(0.7)$$
$$= 6.3$$

The area of the industrial zone is 6.3 km^2.

Key Concepts

- The scale factor, k, relates the lengths of corresponding sides of similar figures. For example, in $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = k$$



$\triangle ABC \sim \triangle PQR$

These relationships can also be written as follows:
$AB = k(PQ)$ $\quad BC = k(QR)$ $\quad CA = k(RP)$

- The square of the scale factor relates the areas of two similar figures:

$$\frac{A_{\Delta ABC}}{A_{\Delta PQR}} = k^2$$

This relationship can also be written as $A_{\triangle ABC} = k^2(A_{\triangle PQR})$.

Communicate Your Understanding

C1 **a)** Explain how the scale factor relates two similar triangles.

b) Explain how you can use the scale factor to find an unknown side length.

C2 **a)** How are the areas of two similar figures related?

b) Explain using words and diagrams how you can find the area of a triangle using the area of a similar triangle and the scale factor.

C3 Explain how you can use a metre stick and shadows to measure the height of an inaccessible object, such as a flagpole or a tree.

Practise

1. A right triangle has side lengths 3 cm, 4 cm, and 5 cm.

a) Draw the triangle.

b) A similar triangle has hypotenuse 30 cm long. What is the scale factor?

c) What are the lengths of the legs?

2. Refer to question 1.

a) Find the area of each triangle.

b) How are these areas related?

c) How do the areas help to confirm that the triangles are similar?

3. a) Draw a triangle.

b) Draw a similar triangle using a scale factor of 2.

c) Repeat part b) using a scale factor of 4.

4. Refer to question 3.

a) Measure, as accurately as possible, the base and height of the first triangle. Use this information to find the area of the triangle. Round your answer to the nearest tenth.

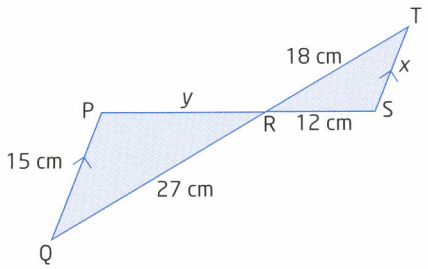
b) Use your answer from part a) and the scale factors to calculate the areas of the two larger triangles.

c) Measure the base and height of each larger triangle and use them to calculate their areas. Compare these results with those obtained in part b). Are they the same? If they are not the same, describe what factors might explain why not.

For help with questions 5 to 7, see Example 1.

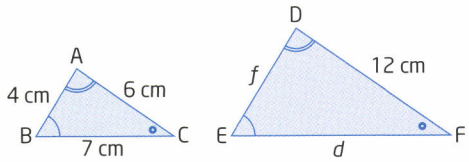
5. a) Show why △PQR is similar to △STR.

b) Find the lengths x and y.

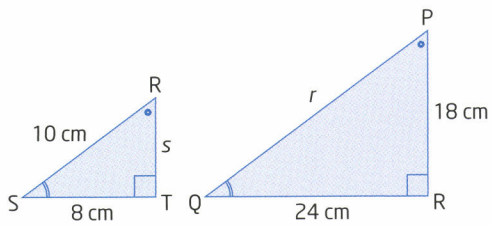


6. The triangles in each pair are similar. Find the unknown side lengths.

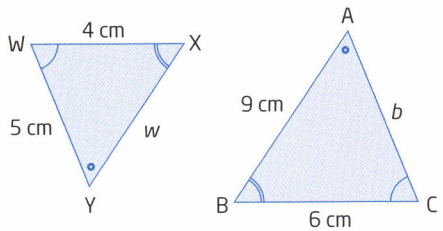
a)



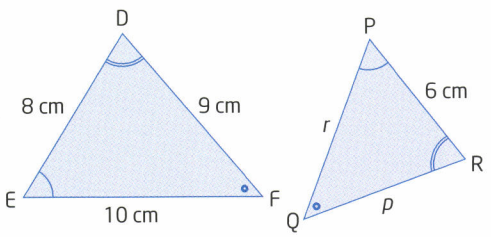
b)



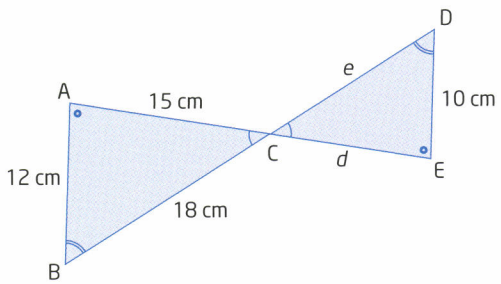
c)



d)

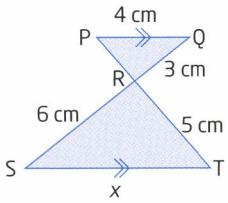


e)

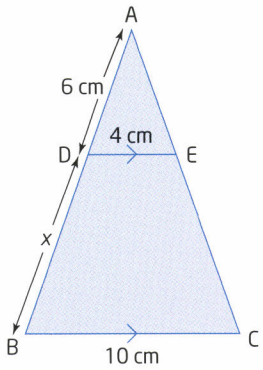


7. Find the length of x in each.

a)

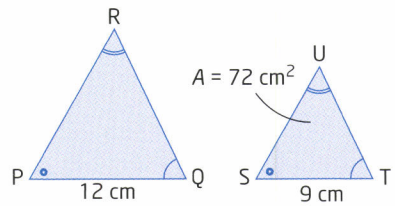


b)

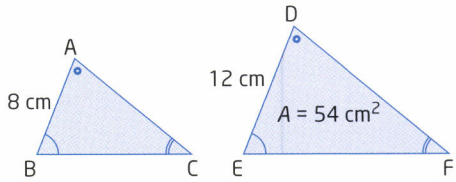


For help with question 8, see Examples 2 and 3.

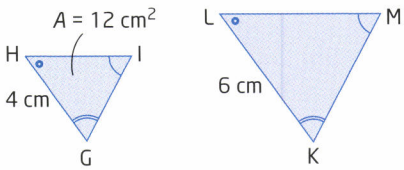
8. a) △PQR ~ △STU. Find the area of △PQR.



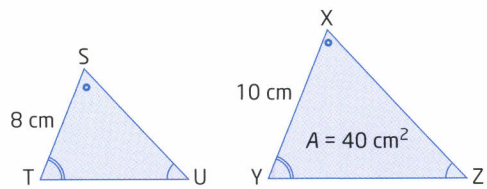
b) △ABC ~ △DEF. Find the area of △ABC.



c) △GHI ~ △KLM. Find the area of △KLM.

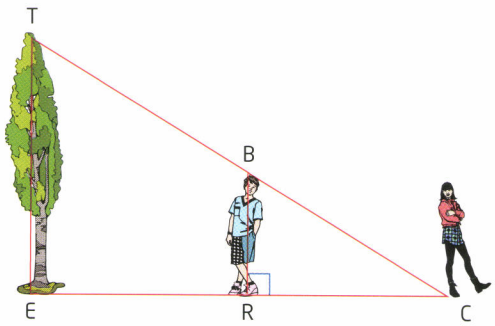


d) △STU ~ △XYZ. Find the area of △STU.



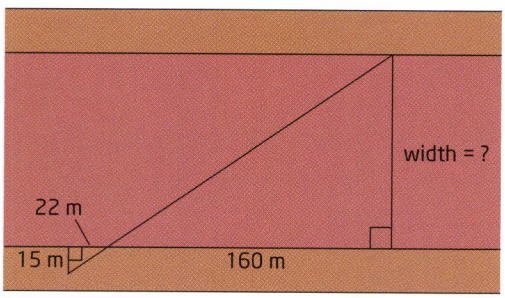
Connect and Apply

9. To measure the height of a tree, Cynthia has her little brother, BR, stand so that the tip of his shadow coincides with the tip of the tree's shadow, at point C.

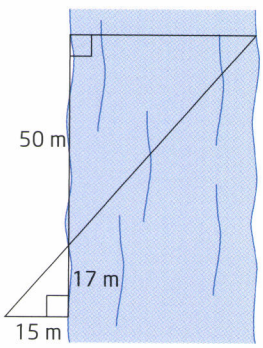


Cynthia's brother, who is 1.2 m tall, is 4.2 m from Cynthia, who is standing at C, and 6.5 m from the base of the tree. Find the height of the tree, TE.

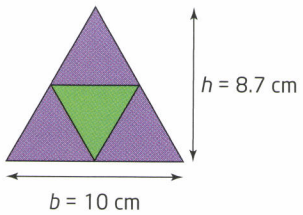
10. Find the width of the canyon.



11. Use the dimensions of the surveyors' triangles to find the width of the river, to the nearest metre.



12. Melanie is designing a crest for her hockey team, the Trigazoids. Her prototype consists of four congruent equilateral triangles.



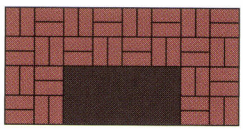
a) What is the total area of this crest?

b) What is the area of
- the green section?
- the purple sections?

c) What is the area of a giant similar crest with base 30 cm?

d) What is the height of a similar crest with area 500 cm^2?

13. The front of each brick in the fireplace measures 10 cm by 20 cm.



a) How many similar rectangles of different sizes can you find? Sketch a diagram to illustrate them. Label their dimensions (length and width).

b) What is the area of the front of one brick?

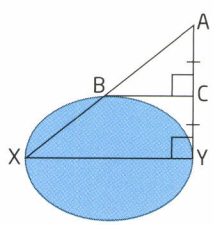
c) Find the area of the entire fireplace, including the opening.

d) Find the area of the opening.

e) Find the area of the fireplace, excluding the opening.

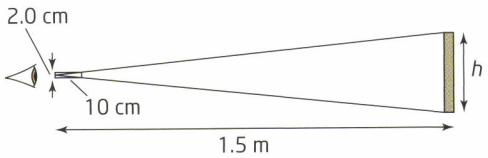
14. Find the length and width of the pond. The following measures are known:
AB = 14 m
BC = 11 m

Assume that XY is a line of symmetry for the pond.



15. Determine the height of a tall tree, a flagpole, or the side of a building in your schoolyard using similar triangles. Explain your method using words and diagrams.

16. While looking through a cylindrical tube, Rita moves to a point where the height of a picture just fits within her field of view, as shown.



Rita is standing 1.5 m from the picture. The length and diameter of the viewing tube are as shown. Find the height of the picture.

17. Use algebraic and geometric reasoning to show how the areas of two similar right triangles are related by the square of the scale factor, k^2.

18. a) Sketch several pairs of similar acute triangles with different scale factors, k.

b) Find the areas of the triangles in each pair.

c) Find the ratio of the areas of the triangles in each pair. How is this ratio related to the scale factor, k?

19. The areas of two similar triangles are 72 cm^2 and 162 cm^2. What is the ratio of the lengths of their corresponding sides?

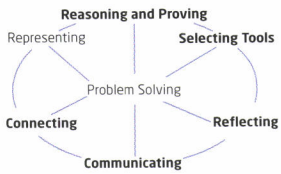
20. $\triangle ABC$ and $\triangle DEF$ are similar. The ratio of their corresponding sides is 3:5. What is the ratio of their perimeters? Explain.

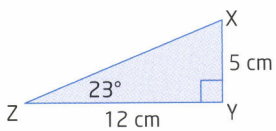
21. Use similar triangles to measure the height of the building in which you live. Write a brief report on how you solved this problem. Include diagrams. Discuss how accurate you think your answer is. Suggest ways to improve your method to get a more accurate height.

22. Chapter Problem The first leg of your race will begin on the southern shore of James Bay, at Moosonee. From there you will travel to Regina, then to Churchill, located on the eastern shore of Hudson Bay. Take note of your journey. The triangle formed by these three locations is similar to the triangle formed by Pittsburgh, Repulse Bay (located near the Arctic Circle), and your next destination. Identify the similar triangles and determine your next destination. *Hint: Move quickly, and you will be glad that you beat the rest of the flock!*

Achievement Check

23. Teschia is making a scale drawing to help her redesign her flower garden.





a) Calculate the length of ZX and the measure of $\angle ZXY$.

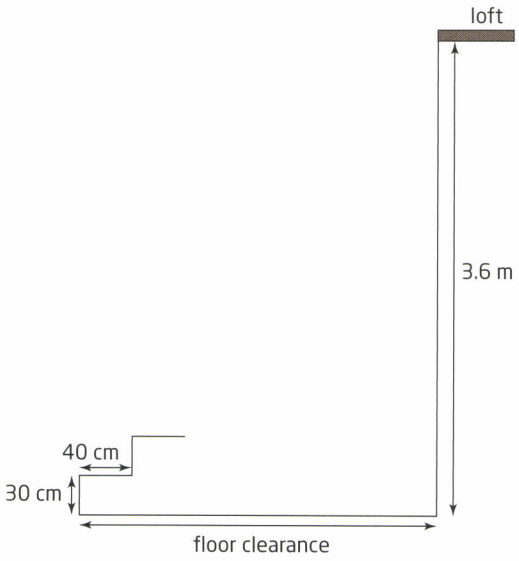
b) If the hypotenuse of the actual flower garden measures 6.5 m, what is the perimeter of the actual garden?

c) What is the scale factor of Teschia's drawing?

d) What is the ratio of the area of the flower garden to the area of the scale drawing?

Extend

24. Carol is building a staircase from the floor of her barn to the loft, which is 3.6 m above the floor. She is using steps that are each 30 cm high and 40 cm deep.



a) How much floor clearance will Carol need in order to fit the staircase?

b) How many steps will be required?

25. The scale on a map is 1 cm represents 5 km. A provincial park has an area of 6 cm^2 on the map. What is the actual area of the park, to the nearest square kilometre?

26. Krista used her Global Positioning System (GPS) device to obtain information on the distance and direction from Niagara Falls to London, England, and to Miami, Florida. She drew a triangle, and calculated the angles in the triangle from the GPS data. She noticed that the sum of the angles was not 180°, as expected.

a) Why did this occur?

b) Would you expect the sum to be more or less than 180°? Explain.

27. Math Contest A naturalist's study in Northern Ontario finds that 25% of the area is water and 60% of the remaining area is forest. The rest, 12 000 ha, is rock. How large is the study area, in hectares?

A 36 000 ha

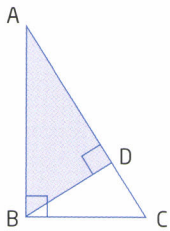
B 40 000 ha

C 68 000 ha

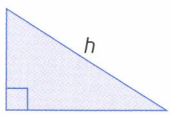
D 80 000 ha

E 100 000 ha

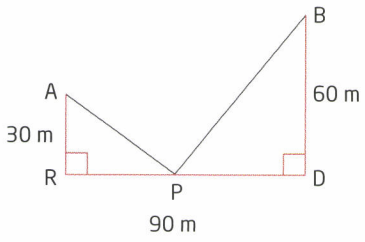
28. Math Contest In $\triangle ABC$, AB = 24 cm and BC = 10 cm. BD is perpendicular to AC. Find the ratio of the shaded area to the unshaded area.



29. Math Contest Express the length of the hypotenuse of a right triangle in terms of its area, A, and it perimeter, P.



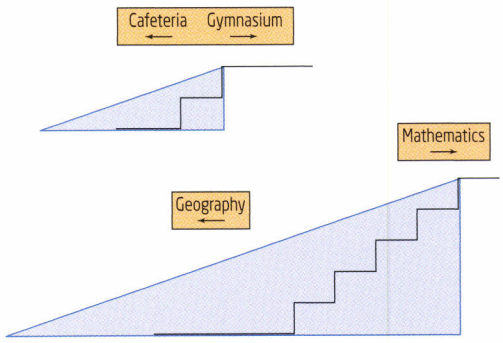
30. Math Contest Two neighbouring houses are located at A and B, near a straight section of a rural road, RD. The electric company plans to place a pole, P, at the roadside and connect wires from the pole to the two houses. How far from point R should the pole be located so that the minimum length of wire is needed?



7.3 The Tangent Ratio

Cory is installing wheelchair ramps at a high school. Not all locations require the same vertical climb, so he will need to adjust the length of the ramp in each case. In general, a wheelchair ramp should have a slope of not more than $\frac{1}{12}$.

How can Cory ensure that all ramps have the same slope? How is the slope related to the angle the ramp makes with the floor?



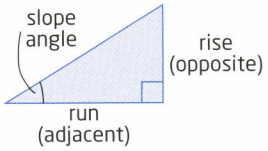
Investigate

Tools

- grid paper
- protractor
- ruler

slope angle

- angle opposite the rise and adjacent to the run



What is the tangent ratio and how is it related to slope?

Method 1: Use Pencil and Paper

1. Draw three similar right triangles on grid paper for each **slope angle**. Each triangle represents a different ramp.
   - 10°
   - 25°
   - 60°

2. **a)** Measure the rise and run of each triangle, and use these to calculate the slope. Record your results in a table like this one.

Triangle	Slope Angle	Rise	Run	Slope (to three decimal places)

b) What do you notice about the slope of the ramp for similar right triangles?

3. The slope in your table in step 2 is called the **tangent** of the slope angle.

   a) Draw three more similar ramps with a different slope angle than those in step 1. Measure the rise, run, and slope of these ramps and record your results in your table. Explain what you notice.

   b) Add a column to your table labelled **Tangent** and use a calculator to calculate the tangent ratio of the slope angle. What do you notice?

4. How can you find the tangent of the other acute angle in the triangle? Rotate each original triangle 90° counterclockwise and repeat the investigation.

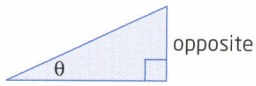
5. **Reflect**

   a) What is the tangent ratio of an angle? What is it the same as?

   b) How does the tangent of an angle change when you change the size of the triangle but keep the angle the same?

tangent of an angle

- the ratio of the side opposite an angle to the side adjacent to the angle



$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Literacy Connections

The short form for tangent is tan. The Greek letter θ, pronounced "theta," is often used to represent angles.

Method 2: Use *The Geometer's Sketchpad*®

1. Open *The Geometer's Sketchpad*® and begin a new sketch.

2. Construct a small ramp.
   - Construct a short horizontal line segment.
   - Select the segment and the right endpoint.
   - From the **Construct** menu, choose **Perpendicular Line**.
   - From the **Construct** menu, choose **Point on Perpendicular Line**.
   - Select the perpendicular line. From the **Display** menu, choose **Hide Perpendicular Line**.
   - Construct segments to complete the right triangle.

3. a) Set the precision of the measurements to thousandths. From the **Edit** menu, choose **Preferences**. Set the **Precision** to **thousandths** for **Angle**, **Distance**, and **Other**.

   b) Measure the rise and run of the ramp, and use these to calculate the slope.
   - Select the vertical and horizontal segments. From the **Measure** menu, choose **Length**.
   - From the **Measure** menu, choose **Calculate**. A calculator will appear.
   - Select the rise measure.
   - Click on ÷.
   - Select the run measure.
   - Click **OK**.

   c) A ratio measure will appear. What does this ratio represent?

   d) Right click on the measure and choose **Label Measurement** to rename the measure to correspond to your answer to part c).

Tools

- computer with *The Geometer's Sketchpad*®

Technology Tip

- Holding the **Shift** key as you draw a segment keeps it horizontal or vertical.
- From the **Display** menu, use the **Hide** command, or press **Ctrl + H**, to tidy up your sketch.

Technology Tip

In step 3, you created a dynamic calculation that automatically updated itself when you changed your sketch.

4. Measure the angle the ramp makes with the floor.

- Select the three points in order so that you select the point where the ramp meets the floor second.
- From the **Measure** menu, choose **Angle**.

Label this measure the **slope angle**.

5. a) Click and drag each vertex of the triangle. Explain what happens in each case.

b) Does the triangle remain a right triangle? Explain why or why not.

c) Return the triangle to the way you had it at the end of step 3.

6. a) Record how the measures change as you change the triangle. To make a table in *The Geometer's Sketchpad®*:

- Select the measurements in the following order: slope angle, rise, run, slope.
- From the **Graph** menu, choose **Tabulate**.
- Right click on the table and choose **Add Table Data**.
- Choose **Add 10 Entries As Values Change**.

Click and drag the vertex above the right angle to change the slope angle. Ten sets of measurements will be recorded in your table.

b) Discover what happens to the slope if you change the size of the triangle but keep the slope angle. Make a new table, as in part a). Click on and drag the vertex at the right angle to make similar triangles with the same slope angle. Record your results in your new table. Describe what happens to each of the following measures:

- rise
- run
- slope
- slope angle

7. Repeat step 6 for a different slope angle. Does the slope of the ramp change if the angle remains the same? Explain.

8. The slope in your table in steps 6 and 7 is called the **tangent** of the slope angle.

You can use *The Geometer's Sketchpad®* to calculate the tangent of an angle.

- From the **Measure** menu, choose **Calculate**.
- Click on the **Functions** key. From the menu of functions, click on **tan**.
- Select the angle measure of the ramp.
- Click **OK**.

Calculate the tangents of several slope angles and compare them to the slope of the ramp. What do you notice?

9. How can you find the tangent of the other acute angle in the triangle? Rotate your original triangle 90° counterclockwise and repeat the investigation.

10. **Reflect**

a) What is the tangent ratio of an angle? What is it the same as?

b) How does the tangent of an angle change when you change the size of the triangle but keep the angle the same?

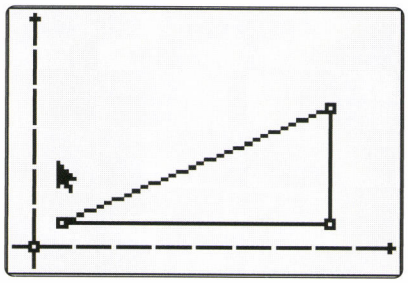
Method 3: Use a Graphing Calculator

Tools

- TI-83 Plus or TI-84 Plus graphing calculator

1. Press APPS and choose **Cabri Jr**. If the axes are not visible, choose the **F5** menu. Choose **Hide/Show**, and then **Axes**. Move the cursor to the origin. Press ALPHA and move the origin until the first quadrant occupies most of the screen, as shown.

2. Construct a small ramp.

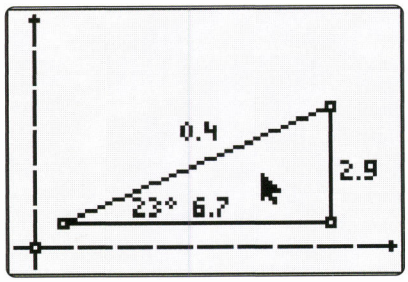


- Choose the **Segment** tool from the **F2** menu. Draw a horizontal line segment, as shown.
- Choose **Perp.** from the **F3** menu.
- Move the cursor to the right endpoint of the segment. Press ENTER. Move the cursor to the segment until it flashes. Press ENTER. A line perpendicular to the segment appears, through the right endpoint.
- Choose **Point** from the **F2** menu, and then **Point on**. Move the cursor to a point on the perpendicular line, and press ENTER.
- Choose **Hide/Show** from the **F5** menu. Then, choose **Object**. Move the cursor to the perpendicular line until it flashes. Press ENTER. The line is hidden.
- Construct segments to complete the right triangle.

3. **a)** Measure the rise and run of the ramp, and use these to calculate the slope.
   - Choose **Measure** from the **F5** menu. Then, choose **D. & Length**.
   - Move the cursor to the run until it flashes. Press ENTER. Move the measurement to a suitable location, and press ENTER again.
   - Repeat this procedure to measure the rise.
   - Choose **Calculate** from the **F5** menu. Move the cursor to the rise measurement, and press ENTER. Move the cursor to the run measurement. Press ENTER. Then, press ÷. Move the calculation to a convenient location, and press ENTER.

b) What does this ratio represent?

4. Measure the angle the ramp makes with the floor.
   - Choose **Measure** from the **F5** menu. Then, choose **Angle**.
   - Move the cursor to the top vertex of the triangle. Press ENTER. Move to the left vertex. Press ENTER. Move to the third vertex. Press ENTER. Move the measurement to a convenient location, and press ENTER.



   Call this the **slope angle**.

5. **a)** Move the cursor to one of the vertices of the triangle. Press ALPHA. Drag the vertex of the triangle. Explain what happens. Try dragging the other vertices, one at a time. Explain what happens.

   b) Does the triangle remain a right triangle? Explain why or why not.

   c) Return the triangle to the way you had it at the end of step 4.

6. **a)** Select and drag the vertex above the right angle to change the slope angle. Record the measures in a table like this one.

Slope Angle	Rise	Run	Slope (to three decimal places)

   b) Discover what happens to the slope if you change the size of the triangle but keep the slope angle the same. Select and drag the left vertex of the triangle. Record your results in the table from part a). Describe what happens to each of the following measures:
   - rise
   - run
   - slope
   - slope angle

7. Repeat step 6 for a different slope angle. Does the slope of the ramp change if the angle remains the same? Explain.

8. The slope in your table in steps 6 and 7 is called the **tangent** of the slope angle. Calculate the tangent of several slope angles and compare it to the slope of the ramp. What do you notice?

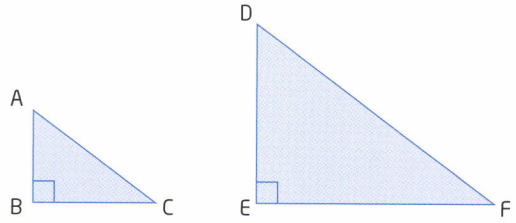
9. How can you find the tangent of the other acute angle in the triangle? Rotate your original triangle 90° counterclockwise and repeat the investigation.

10. **Reflect**

   a) What is the tangent ratio of an angle? What is it the same as?

   b) How does the tangent of an angle change when you change the size of the triangle but keep the angle the same?

In the Investigate, you saw that the tangent of an acute angle in a right triangle is constant if the angle stays the same, even if the size of the triangle changes. This is because of the properties of similar triangles. Since the ratios of corresponding sides of similar triangles are equal, and the tangent is a ratio of sides, the tangents of the angles of similar triangles are equal. For example, in similar triangles $\triangle ABC$ and $\triangle DEF$,



$$\tan C = \frac{\text{opposite}}{\text{adjacent}} \quad \text{and} \quad \tan F = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{AB}{BC} \qquad\qquad = \frac{DE}{EF}$$

By similar triangles,

$$\frac{AB}{DE} = \frac{BC}{EF}$$
$$\frac{AB}{BC} = \frac{DE}{EF}$$

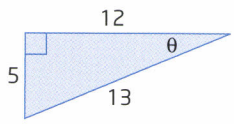
Divide both sides by BC and multiply both sides by DE.

So, $\tan C = \tan F$.

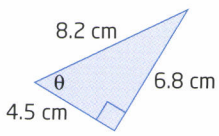
Example 1 Find the Tangent Ratio From Given Sides

Find $\tan \theta$ for each triangle, expressed as a fraction and as a decimal correct to four decimal places.

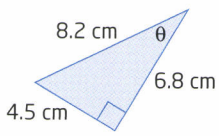
a)



b)



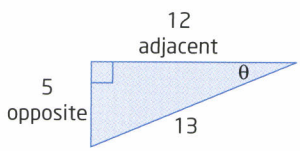
c)



Solution

a) Identify the opposite and adjacent sides relative to the angle θ.

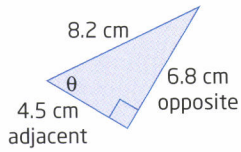
$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{5}{12}$$
$$\doteq 0.4167$$



Literacy Connections

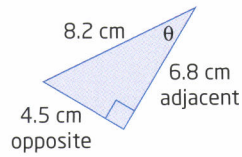
Expressions involving tangent mean "the tangent of." For example, tan A means "the tangent of angle A." Similarly, tan 25° means "the tangent of 25°."

b) Identify the opposite and adjacent sides.



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{6.8}{4.5}$$
$$\doteq 1.5111$$

c) When you go from one acute angle to the other in a right triangle, the opposite and adjacent sides relative to the angle become reversed.



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{4.5}{6.8}$$
$$\doteq 0.6618$$

This is the same triangle as in part b), but I am calculating the tangent of the other acute angle.

Example 2 Find the Tangent of an Angle

Evaluate each of the following with a calculator. Record your answer rounded to four decimal places.

a) tan 25°

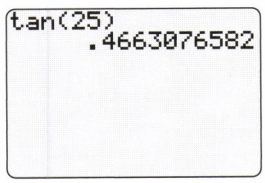
b) tan 60°

Solution

Use a scientific or graphing calculator.

a) tan 25° $\doteq$ 0.4663

b) tan 60° $\doteq$ 1.7321



Technology Tip

Not all scientific calculators work the same way. With some, you press
TAN 25 =

With others, you press
25 TAN

On a graphing calculator, press
TAN 25) =

Make sure your calculator is in degree mode.

Calculators have an inverse function that allows you to apply the tangent ratio in reverse. If you know the ratio $\frac{\text{opposite}}{\text{adjacent}}$, you can find the angle whose tangent this ratio represents.

Example 3 Find an Angle Using the Tangent Ratio

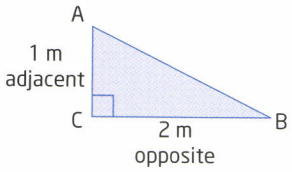
Fiona is building a skateboarding ramp. She wants the ramp to rise 1 m in a horizontal distance of 2 m. At what acute angles should she cut the wood, rounded to the nearest degree?

Solution

Sketch and label a diagram of the ramp.

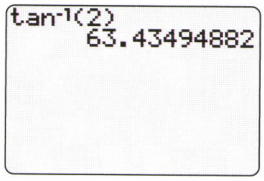
Find $\angle A$.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{2}{1}$$
$$= 2$$



To find $\angle A$, calculate the inverse tangent of 2.

$\angle A = \tan^{-1}(2)$
$\doteq 63.43$

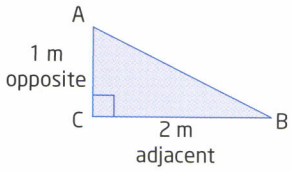


One of the acute angles is about 63°.

Find $\angle B$.

Method 1: Apply the Tangent Ratio

$$\tan B = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{1}{2}$$
$$= 0.5$$



Apply the inverse tangent operation.

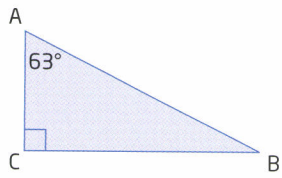
$\angle B = \tan^{-1}(0.5)$
$\doteq 26.56$

The other acute angle is about 27°.

Method 2: Use Geometric Reasoning

The sum of three interior angles in any triangle is 180°.

$\angle A + \angle B + \angle C = 180°$
$63° + \angle B + 90° = 180°$
$\angle B = 180° - (63° + 90°)$
$= 180° - 153°$
$= 27°$



The other acute angle is about 27°.

Fiona should cut the wood using acute angles of approximately 63° and 27°.

Note that Method 1 will give a more accurate measure for the angle, since only given information is being used.

Literacy Connections

Tangent and **inverse tangent** are opposite operations, like addition and subtraction. For example,

$\tan 60° \doteq 1.7321$

$\tan^{-1}(1.7321) \doteq 60°$

The second statement is read as "the inverse tangent of 1.7321 is approximately equal to 60 degrees."

Technology Tip

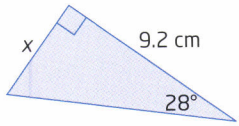
With some scientific calculators, you press
2nd [TAN^{-1}] 0.5 =

With others, you press
0.5 2nd [TAN^{-1}]

The tangent ratio relates two sides of a right triangle and an angle. If you know an angle and the length of one of the legs of the triangle, you can find the length of the other leg.

Example 4 Find a Side Length Using the Tangent Ratio

Find the length, x, in the diagram, rounded to the nearest tenth of a centimetre.



Solution

Write the tangent ratio for the given angle in terms of the side lengths.

$$\tan 28° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 28° = \frac{x}{9.2}$$

$9.2(\tan 28°) = x$ Multiply both sides by 9.2.

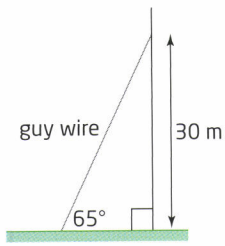
$4.891 \doteq x$ 9.2 × 28 TAN = or 9.2 × TAN 28 =

The length of side x is about 4.9 cm.

Example 5 Solve a Multi-Step Problem Using the Tangent Ratio

A radio transmitter is to be supported with a guy wire, as shown. The wire is to form a 65° angle with the ground and reach 30 m up the transmitter.

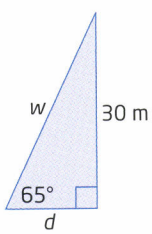
The wire can be ordered in whole-number lengths of metres. How much wire should be ordered?



Solution

Use the tangent ratio of the given angle to find the distance, d, from the tower that the guy wire should be secured.

Then, apply the Pythagorean theorem to find the length, w, of wire needed.



$$\tan 65° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 65° = \frac{30}{d}$$

$d(\tan 65°) = 30$ Multiply both sides by d.

$$d = \frac{30}{\tan 65°}$$ Divide both sides by tan 65°.

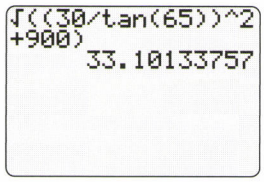
Apply the Pythagorean theorem to find the length of wire needed.

$w^2 = d^2 + 30^2$

$$w^2 = \left(\frac{30}{\tan 65°}\right)^2 + 900$$

$$w = \sqrt{\left(\frac{30}{\tan 65°}\right)^2 + 900}$$

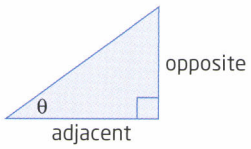
$w \doteq 33.101$



The length of guy wire needed is about 33.1 m. It can only be ordered in whole-number lengths. 33 m is too short, so round up to the next metre. At least 34 m of guy wire should be ordered.

Key Concepts

- The ratio of the opposite side to the adjacent side of an angle in a right triangle is called the tangent of that angle.



$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- You can use a scientific or graphing calculator to
  - express the tangent of an angle as a decimal
  - find one of the acute angles when both leg lengths are known in a right triangle
  - find a side length if one acute angle and one leg of a right triangle are known

Communicate Your Understanding

C1 Explain how the tangent of the slope angle is related to the slope of a ramp.

C2 **a)** Explain how you can identify the opposite and adjacent sides of a right triangle.

b) Explain how these can change in a given triangle. Use a diagram to support your explanation.

C3 **a)** Explain what each calculator function does:
- tangent
- inverse tangent

b) When would you use each function?

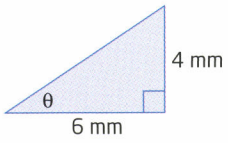
c) How are these functions related to each other?

Practise

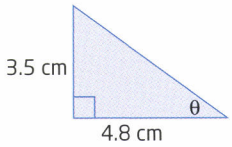
For help with questions 1 and 2, see Example 1.

1. Find the tangent of the angle indicated, to four decimal places.

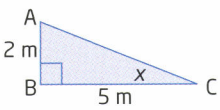
a)



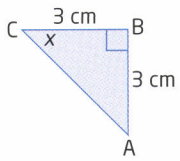
b)



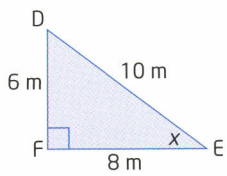
c)



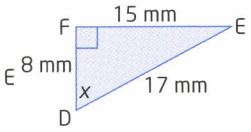
d)



e)



f)



2. Refer to question 1. Find the tangent of the other acute angle, to four decimal places.

For help with question 3, see Example 2.

3. Evaluate with a calculator. Record your answer to four decimal places.

a) tan 65°
b) tan 15°
c) tan 62°
d) tan 5°
e) tan 30.7°
f) tan 82.4°
g) tan 20.5°
h) tan 45°

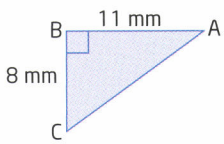
For help with questions 4 to 8, see Example 3.

4. Find the measure of each angle, to the nearest degree.

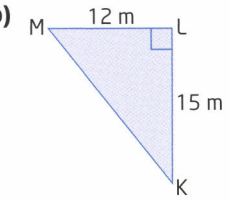
a) $\tan \theta = 1.5$
b) $\tan A = \frac{3}{4}$
c) $\tan B = 0.6000$
d) $\tan W = \frac{4}{5}$
e) $\tan C = 0.8333$
f) $\tan \theta = \frac{6}{7}$
g) $\tan X = 3.0250$
h) $\tan \theta = \frac{15}{9}$

5. Find the measures of both acute angles in each triangle, to the nearest degree.

a)



b)



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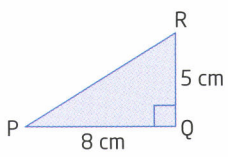

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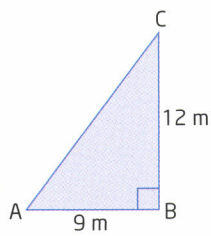
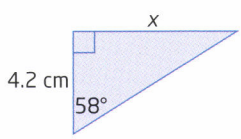
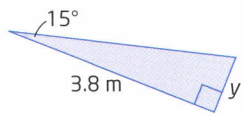


6. Find the length of the unknown side, to the nearest tenth.

a)



b)



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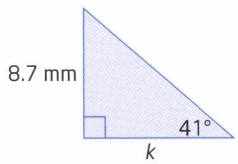

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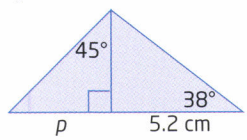
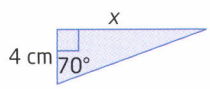
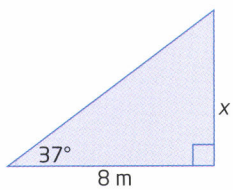


7. Find the length of x, to the nearest tenth of a metre.

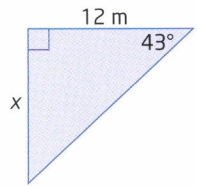
a)



b)

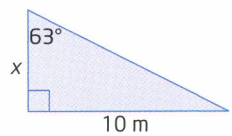


c)

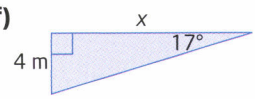


d)

e)

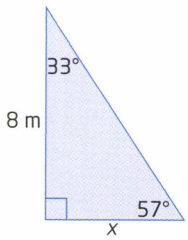


f)

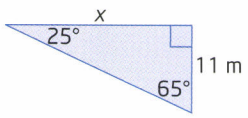


8. Find the length of x, to the nearest tenth of a metre.

a)

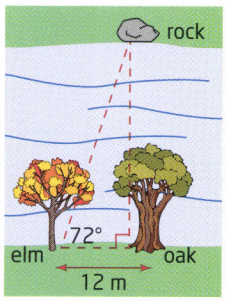


b)



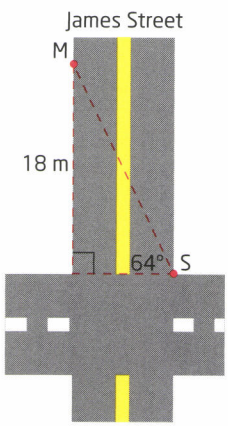
Connect and Apply

9. To measure the width of a river, Kirstyn uses a large rock, an oak tree, and an elm tree, which are positioned as shown.

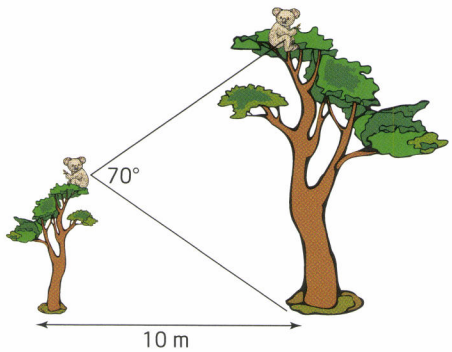


Show how Kirstyn can use the tangent ratio to find the width of the river, to the nearest metre.

10. A surveyor is positioned at a traffic intersection, viewing a marker on the other side of the street. The marker is 18 m from the intersection. The surveyor cannot measure the width directly because there is too much traffic. Find the width of James Street, to the nearest tenth of a metre.



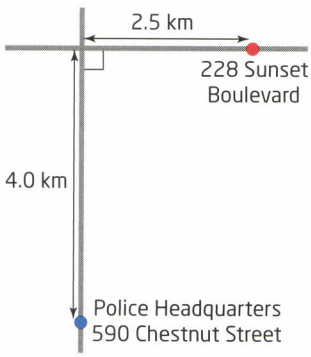
11. Rocco and Biff are two koalas sitting at the top of two eucalyptus trees, which are located 10 m apart, as shown. Rocco's tree is exactly half as tall as Biff's tree. From Rocco's point of view, the angle separating Biff and the base of his tree is 70°.



How high off the ground is each koala?

12. Police are responding to a distress call:

Police headquarters and the trouble site are shown.



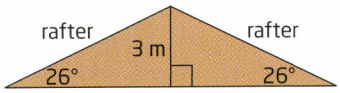
Squad cars and a helicopter are both immediately dispatched to the site from headquarters.

a) At what angle to Chestnut Street should the helicopter travel?

b) Assuming that the squad cars can travel at an average speed of 60 km/h and the helicopter can travel twice as fast, how much longer will it take for the squad cars to reach the site than the helicopter?

c) Describe any assumptions you make in your solutions.

13. The diagram shows the roof of a house. How wide is the house, to the nearest metre?

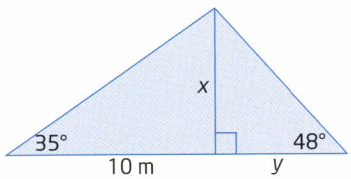


14. Petra walked diagonally across a rectangular schoolyard measuring 45 m by 65 m. To the nearest degree, at what angle with respect to the shorter side did she walk?

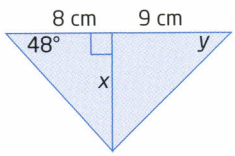
15. Comfortable stairs have a slope of $\frac{3}{4}$.

What angle do the stairs make with the horizontal, to the nearest degree?

16. Find the length of x, then the length of y, to the nearest tenth of a metre.



17. Find the length of x, to the nearest tenth of a centimetre, then the measure of $\angle y$, to the nearest degree.



18. To measure the height of a building, Chico notes that its shadow is 8.5 m long. He also finds that a line joining the top of the building to the tip of the shadow forms a 65° angle with the flat ground.

a) Draw a diagram to illustrate this situation.

b) Find the height of the building, to the nearest tenth of a metre.

19. a) Find the tangent of several angles with values between 1° and 44°. Organize your results in a table like this one.

Angle, θ	tan θ

b) What is the value of tan 45°?

c) Add the tangents of several angles with values between 46° and 89° to your table.

d) Add the tangents of several angles with values very close to, but not equal to, 90° to your table.

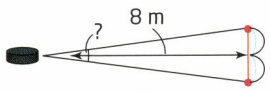
e) Based on your findings, what conclusions can you make about the tangents of angles
- less than 45°?
- greater than 45° and less than 90°?
- very close to, but not equal to, 90°?

f) Use the definition of the tangent ratio and geometric reasoning to justify your conclusions. Include diagrams in your explanation.

20. a) Use a calculator to evaluate the following:
- tan 0°
- tan 90°

b) Use the definition of the tangent ratio and geometric reasoning to explain your results. Include diagrams in your explanation.

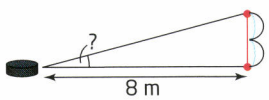
21. At hockey practice, Lars has the puck in front of the net, as shown.



He is exactly 8 m away from the middle of the net, which is 2 m wide. Within what angle must Lars fire his shot in order to get it in the net, to the nearest degree?

22. Refer to question 21.

a) Does Lars have a better chance, a worse chance, or the same chance to score if he positions himself directly in front of one of the posts, as shown? Explain your reasoning and any assumptions you make.



b) Repeat part a) for the case in which Lars moves
- directly closer to the net
- directly farther from the net

Extend

23. **a)** Make a table of values for $\tan \theta$ for values of θ between $0°$ and $90°$.

b) Graph the relationship. Is the relationship linear or non-linear? Explain.

c) Describe the shape of the graph and any interesting features you can identify.

24. The angle θ at which a skier slides down a hill with a coefficient of friction, μ, at a constant speed, is given by $\tan \theta = \mu$. Natalie is skiing on a hill with a reported coefficient of friction of 0.6. If Natalie skis down at a constant speed, what is the angle of the hill?

25. The tangent ratio is used to design the bank angle for a curved section of roadway.

Let θ be the bank angle required for a speed limit, v, in kilometres per hour, and a radius, r, in metres. The angle and the speed limit are related by the formula $\tan \theta = \frac{v^2}{9.8r}$. Find the bank angle required for a highway curve of radius 50 m that will carry traffic moving at 100 km/h.

Did You Know?

This same relation applies to banking a bicycle or motorcycle when going around a curve, and banking an aircraft in a turn.

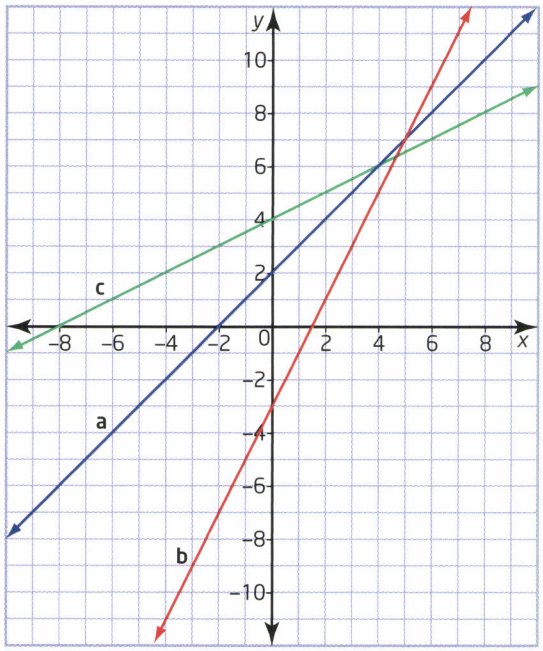
26. For each graph,

i) find the slope of the line

ii) draw a triangle to find the tangent of the acute angle that the line makes with the x-axis

iii) compare your answers to parts i) and ii)

iv) find the acute angle in part ii)



27. **Math Contest** How many numbers less than 10 000 contain at least one 5?

A 5000

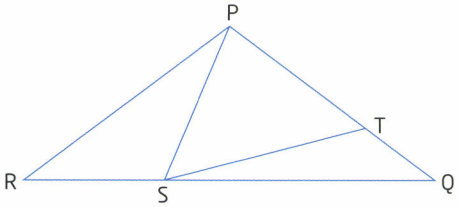
B 6561

C 3439

D 625

E 4944

28. **Math Contest** In the figure, PR = PQ and $\angle RPS = 30°$. IF PS = PT, what is the measure of $\triangle QST$?



7.4 The Sine and Cosine Ratios

We depend on ships and aircraft to transport goods and people all over the world. If you were the captain of a ship or the pilot of an airplane, how could you make sure that you did not get lost in the middle of the ocean? In ancient times, this was a significant problem.

Today, navigational equipment such as Global Positioning System (GPS) devices makes it much easier to find your way around the planet. Even so, factors such as wind and water currents can sometimes complicate travel plans. How can trigonometry help when this happens?

Investigate

Tools

- computer with *The Geometer's Sketchpad®*

OR

- TI-83 Plus or TI-84 Plus graphing calculator

OR

- grid paper
- protractor
- ruler

What are the sine and cosine ratios?

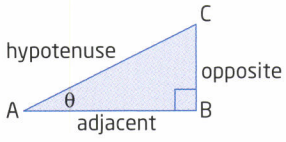
In the last section, you learned the tangent ratio. In this activity, you will investigate two other important ratios. In addition to the opposite and adjacent sides, these ratios involve the third side of a right triangle: the hypotenuse.

1. a) Draw a large right triangle $\triangle ABC$.

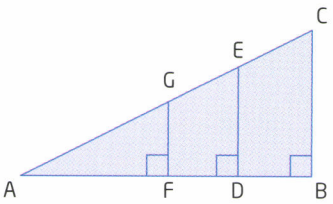
b) Measure the length of the side opposite $\angle A$.

c) Measure the length of the hypotenuse.

d) Calculate the ratio $\dfrac{\text{opposite}}{\text{hypotenuse}}$.



2. a) Create overlapping triangles by adding line segments parallel to one of the legs as shown.



b) Explain why these triangles are similar to the first one.

c) Measure and calculate the following for each similar triangle:

- length of the opposite side
- length of the hypotenuse
- the ratio $\frac{\text{opposite}}{\text{hypotenuse}}$

d) Compare the $\frac{\text{opposite}}{\text{hypotenuse}}$ ratios for each triangle and describe what you notice.

3. a) Measure the length of the sides adjacent to $\angle A$ for each triangle.

b) Calculate the ratio $\frac{\text{adjacent}}{\text{hypotenuse}}$ for each triangle. Describe what you notice.

4. a) Draw a new set of similar triangles, with a different value for $\angle A$.

b) Calculate and compare the ratio of $\frac{\text{opposite}}{\text{hypotenuse}}$ for each similar triangle.

c) Calculate and compare the ratio of $\frac{\text{adjacent}}{\text{hypotenuse}}$ for each similar triangle.

d) Repeat for another set of similar triangles.

The two ratios you have just explored are called the sine and cosine ratios. They are defined as

$$\text{sine } A = \frac{\text{opposite}}{\text{hypotenuse}} \quad \text{and} \quad \text{cosine } A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Together with the tangent ratio, they are the three **primary trigonometric ratios**.

5. Reflect Summarize what you have discovered about the three primary trigonometric ratios, using words and diagrams.

Technology Tip

To do step 2 using dynamic geometry software, follow these steps:

- Select the horizontal line segment. Construct a point on it.
- Select this segment and the new point, and construct a perpendicular line.
- Select the perpendicular line and the hypotenuse. Construct the point of intersection.
- Hide the perpendicular line. Construct a line segment connecting the two new points.

Literacy Connections

The short forms for sine and cosine are sin and cos.

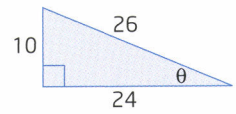
primary trigonometric ratios

- sine, cosine, and tangent
- often abbreviated as sin, cos, and tan

Example 1 Find the Primary Trigonometric Ratios

Find the three primary trigonometric ratios for θ. Express the ratios as decimals, rounded to four decimal places.

a)



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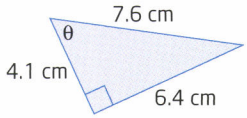
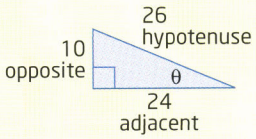


Solution

I need to identify the opposite, adjacent, and hypotenuse sides relative to the angle θ.



a)

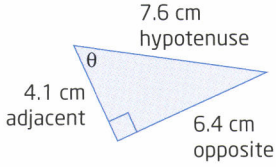
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{10}{26} \qquad = \frac{24}{26} \qquad = \frac{10}{24}$$

$$= \frac{5}{13} \qquad = \frac{12}{13} \qquad = \frac{5}{12}$$

$$\doteq 0.3846 \qquad \doteq 0.9231 \qquad \doteq 0.4167$$

b)



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{6.4}{7.6} \qquad = \frac{4.1}{7.6} \qquad = \frac{6.4}{4.1}$$

$$\doteq 0.8421 \qquad \doteq 0.5395 \qquad \doteq 1.5610$$

Since sine, cosine, and tangent are ratios, they have no units.

Literacy Connections

A memory device (or mnemonic) for the three primary trigonometric ratios uses these short forms:

$S = \frac{O}{H}$ $C = \frac{A}{H}$ $T = \frac{O}{A}$

These short forms produce the nonsense phrase soh cah toa. This phrase may help you remember the formulas for the trigonometric ratios.

Making Connections

Trigonometric ratios are often expressed with three or four digits of accuracy. This is to ensure that angles are found with enough precision. Consider the difference in the following two calculations:

$\tan^{-1}(0.2) \doteq 11.310°$ $\tan^{-1}(0.3) \doteq 16.699°$

Compare the difference between these two angles:

$\tan^{-1}(0.2492) \doteq 13.993°$ $\tan^{-1}(0.2493) \doteq 13.998°$

In both cases, the tangents of the angles differ by one decimal place. But in the second case, the angles are much closer together. Possible discrepancies due to rounding are reduced by carrying more digits until the final step in a calculation.

Just as with the tangent ratio, you can find the sine and cosine of an angle using a scientific or graphing calculator.

Example 2 Find the Sine and Cosine of an Angle

Evaluate the following, to four decimal places.

a) $\sin 26°$ **b)** $\cos 75°$

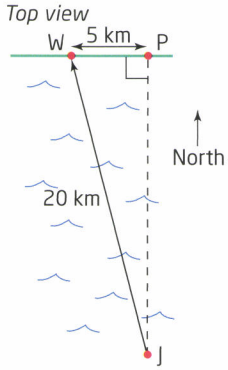
Solution

Make sure that your calculator is in degree mode.

a) $\sin 26° \doteq 0.4384$ **b)** $\cos 75° \doteq 0.2588$

Example 3 Find an Angle Using the Sine and Cosine Ratios

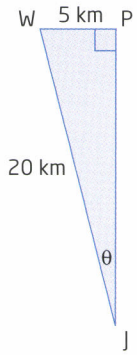
a) Captain Jack is navigating his ship to Port Harbour, which is directly north of the ship's location. To compensate for an easterly current, he aims for a point on shore that is 5 km west of Port Harbour. Assuming that the point on shore is 20 km from his position now, at what bearing must Jack head his ship?



b) Captain Jack is in communication with a submarine that is cruising at a depth of 400 m below sea level. If Jack's radar tells him that the submarine is 500 m from Jack, due north of his ship, at what angle is the submarine located with respect to Captain Jack's ship, to the nearest degree?

Solution

a) Captain Jack's ship, Port Harbour, and the western target form a right triangle.



For the unknown bearing angle, the opposite and hypotenuse sides are known.

Apply the sine ratio to find Jack's bearing.

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$
$$= \frac{5}{20}$$
$$= 0.25$$

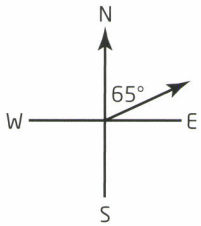
To find θ, calculate the inverse sine of 0.25.

$\theta = \sin^{-1}(0.25)$
$\doteq 14.477°$

Jack must head his ship on a bearing of approximately N15°W.

Literacy Connections

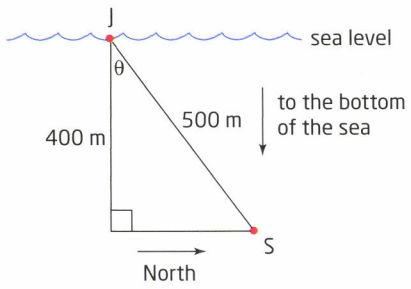
Bearing is a navigational term that describes a direction. It is expressed as an angle in terms of north, south, east, and/or west.



This bearing can be described as N65°E, which is read as "65 degrees east of north."

b) Draw a diagram that shows the relative positions of Captain Jack's ship and the submarine. Captain Jack's ship, the submarine, and a line segment that points straight down from Captain Jack's ship form a right triangle.

Side view, facing west



For the unknown angle relating the submarine's position to Captain Jack, the adjacent and hypotenuse sides are known. Apply the cosine ratio to find this angle.

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{400}{500}$$

$$\theta = \cos^{-1}\left(\frac{400}{500}\right)$$

$$\doteq 36.870$$

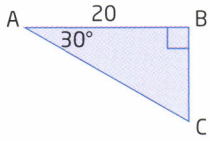
2nd [COS⁻¹] (400 ÷ 500) =

or (400 ÷ 500) 2nd [COS⁻¹]

The submarine is approximately 37° north of Captain Jack's ship with respect to a line that points straight down to the bottom of the sea.

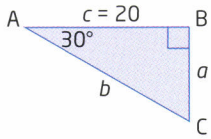
Example 4 Solve a Right Triangle

Solve △ABC. Round side lengths to the nearest unit and angles to the nearest degree.



Solution

Label the sides according to their corresponding angles.



Use the two known angles to find ∠C.

$\angle A + \angle B + \angle C = 180°$

$30° + 90° + \angle C = 180°$

$120° + \angle C = 180°$

$\angle C = 180° - 120°$

$\angle C = 60°$

Note that in a right triangle, the two acute angles are complementary. You can find ∠C by subtracting: $90° - 30° = 60°$.

Literacy Connections

A triangle has six measures: three side lengths and three angle measures. To solve a triangle means to find all six of these values.

Literacy Connections

When naming parts of triangles, use capital letters to represent the angles and vertices and corresponding lowercase letters to represent opposite sides.

Use the cosine ratio to find side b.

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{c}{b}$$

$$\cos 30° = \frac{20}{b}$$

$b(\cos 30°) = 20$ **Multiply both sides by b.**

$$b = \frac{20}{\cos 30°}$$ **Divide both sides by cos 30°.**

$b \doteq 23.094$ **20 ÷ COS 30 =** or **20 ÷ 30 COS =**

Use the tangent ratio to find a.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

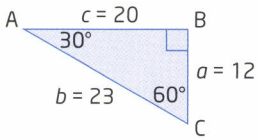
$$\tan A = \frac{a}{c}$$

$$\tan 30° = \frac{a}{20}$$

$20(\tan 30°) = a$ **Multiply both sides by 20.**

$a \doteq 11.547$ **20 × TAN 30 =** or **20 × 30 TAN =**

The diagram shows the solved triangle.



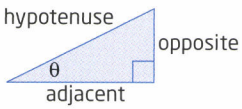
There is often more than one way to solve a right triangle. If you know the value of any side plus an additional side or angle, you can find the other measures. Notice, however, that some answers may be slightly different, due to rounding in the intermediate steps of a solution.

Key Concepts

- The three primary trigonometric ratios are sine, cosine, and tangent. They are defined as follows:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

- You can find any side length or angle measure of a right triangle if you know two pieces of information in addition to the right angle.



Communicate Your Understanding

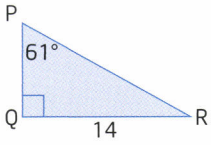
C1 Explain why the primary trigonometric ratios depend only on a given angle and not the size of a right triangle.

C2 **a)** Create a problem for which you would need to apply the cosine function on your calculator. Solve the problem and explain each step.

b) Repeat part a) for the inverse sine function of your calculator.

C3 **a)** List the steps you would use to solve $\triangle PQR$.

b) List a different set of steps to solve $\triangle PQR$ using another method.

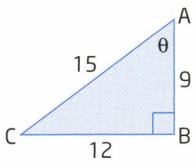


Practise

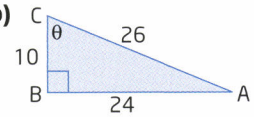
For help with questions 1 and 2, see Example 1.

1. Find $\sin\theta$, $\cos\theta$, and $\tan\theta$ for each triangle, expressed as fractions in lowest terms.

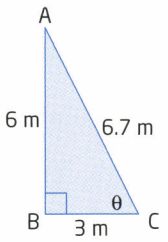
a)



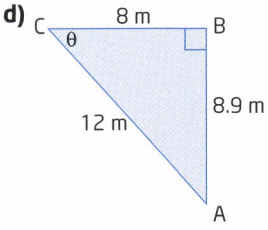
b)



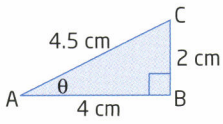
c)



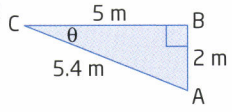
d)



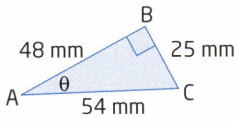
e)



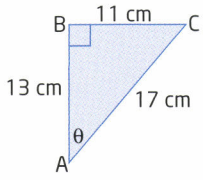
f)



g)

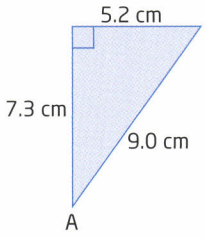


h)

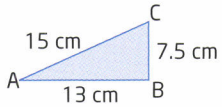


2. Find the three primary trigonometric ratios for $\angle A$, to four decimal places.

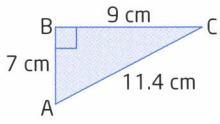
a)



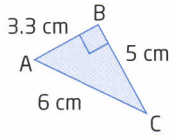
b)



c)



d)



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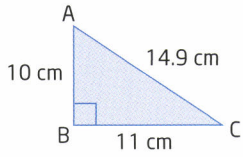
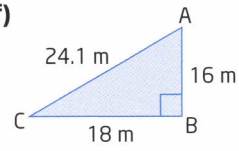


f)



For help with questions 3 to 5, see Example 2.

3. Evaluate each of the following with a calculator, rounded to four decimal places.

a) $\sin 35°$ **b)** $\sin 45°$

c) $\sin 60°$ **d)** $\sin 37°$

e) $\sin 25°$ **f)** $\sin 0°$

g) $\sin 89°$ **h)** $\sin 30°$

4. Evaluate each of the following with a calculator, rounded to four decimal places.

a) $\cos 80.2°$ **b)** $\cos 45°$

c) $\cos 30°$ **d)** $\cos 60°$

e) $\cos 89°$ **f)** $\cos 0°$

g) $\cos 5°$ **h)** $\cos 83°$

5. Compare your results to questions 3 h) and 4 d). Use a diagram to help explain these results.

For help with questions 6 to 9, see Example 3.

6. Find the measure of each angle, to the nearest degree.

a) $\sin \theta = 0.8933$ **b)** $\sin \theta = 0.5032$

c) $\sin P = \frac{1}{2}$ **d)** $\sin S = \frac{2}{3}$

e) $\sin \theta = \frac{3}{4}$ **f)** $\sin A = 0.9511$

g) $\sin \theta = 0.7123$ **h)** $\sin \theta = \frac{2}{5}$

i) $\sin X = 0.3035$ **j)** $\sin \theta = 0.9976$

k) $\sin V = \frac{1}{8}$ **l)** $\sin \theta = 0$

7. Find the measure of each angle, to the nearest degree.

a) $\cos \theta = 0.4481$ **b)** $\cos A = 0.6329$

c) $\cos C = \frac{5}{11}$ **d)** $\cos \theta = 0.3432$

e) $\cos Q = 0.8871$ **f)** $\cos M = \frac{3}{14}$

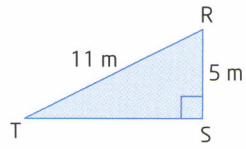
g) $\cos \theta = \frac{1}{6}$ **h)** $\cos \theta = 0.6215$

i) $\cos B = \frac{15}{16}$ **j)** $\cos X = 0.0193$

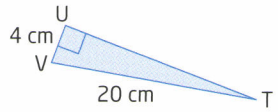
k) $\cos \theta = 0$ **l)** $\cos J = \frac{1}{2}$

8. Calculate sin T in each triangle. Then, find $\angle T$, to the nearest degree.

a)

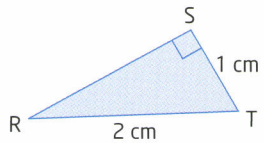


b)

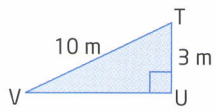


9. Calculate cos T in each triangle. Then, find $\angle T$, to the nearest degree.

a)



b)

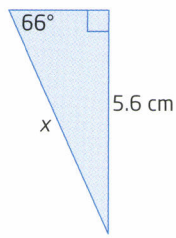


For help with questions 10 to 14, see Example 4.

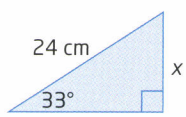
10. Find the length of x, to the nearest tenth of a unit, by applying the sine ratio.

a)

b)



c)



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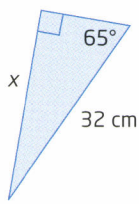

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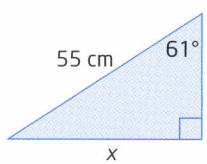

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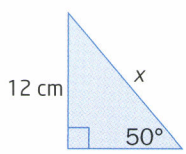

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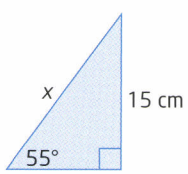
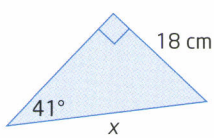
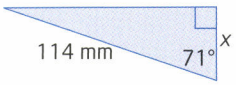


h)

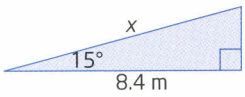


11. Find the length of x, to the nearest tenth of a unit, by applying the cosine ratio.

a)



b)



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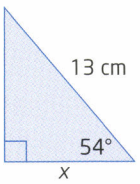

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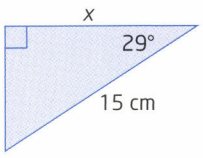

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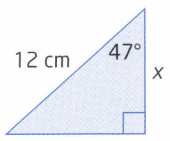

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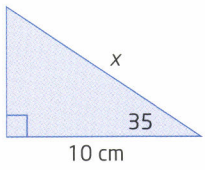
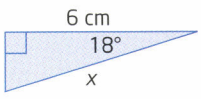


g)



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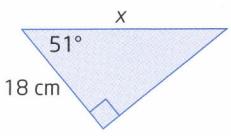
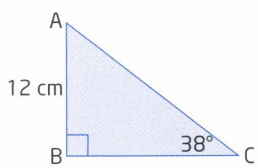


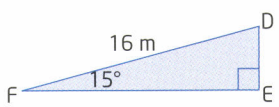
In questions 12 to 14, round side lengths to the nearest tenth of a unit and angles to the nearest degree.

12. Solve each triangle.

a)



b)



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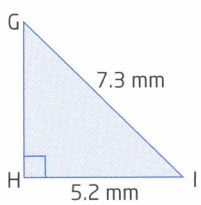
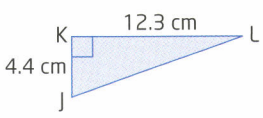


d)



13. In $\triangle DEF$,
$DF = 6.0$ km
$\angle E = 44°$
$\angle F = 90°$

a) Draw this triangle and label the given information.

b) Solve $\triangle DEF$.

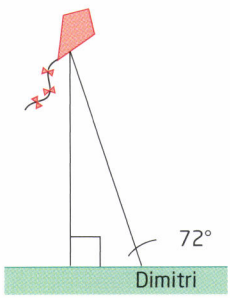
14. In $\triangle XYZ$,
$XY = 16$ cm
$YZ = 11$ cm
$\angle Z = 90°$

a) Draw this triangle and label the given information.

b) Solve $\triangle XYZ$.

Connect and Apply

15. Dmitri has let out 40 m of his kite string, which makes an angle of 72° with the horizontal ground.



a) Find the height of the kite, to the nearest metre.

b) Suppose the Sun is shining directly above the kite. How far is the kite's shadow from Dmitri, to the nearest metre?

16. During take-off a plane must rise at least 20 m during its first 1.5 km of flight to successfully clear the runway.

a) At what minimum average angle must the plane climb for a safe take-off, to the nearest hundredth of a degree?

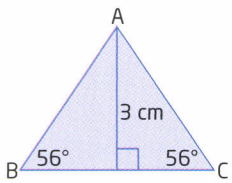
b) If the required rise is doubled to 40 m, does this double the climb angle? Explain.

17. In $\triangle PQR$, $\angle Q = 90°$ and $PR = 20$ cm. Find PQ, to the nearest tenth of a centimetre, if $\angle R = 41°$.

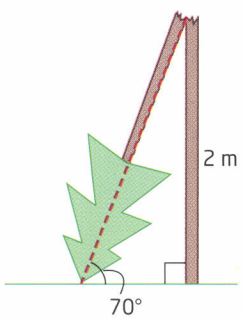
18. In $\triangle DEF$, find $\angle F$, to the nearest degree, if $DE = 15$ cm, $DF = 18$ cm, and $\angle E = 90°$.

19. The towrope pulling a parasailor is 70 m long. A boat crew member estimates that the angle between the towrope and the water is about 30°. Find the height of the parasailor above the water.

20. $\triangle ABC$ is an isosceles triangle. The height of the triangle is 3 cm, and the two acute angles at its base are each 56°. How long are the two equal sides, to the nearest tenth of a centimetre?



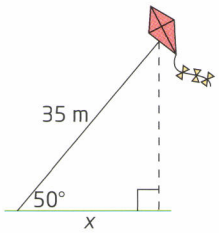
21. A tree is splintered by lightning 2 m up its trunk, so that the top part of the tree touches the ground. The angle the top of the tree forms with the ground is 70°. Before it was splintered, how tall was the tree, to the nearest tenth of a metre?



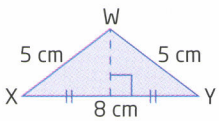
22. The side adjacent to the 74° angle in a right triangle is 6 cm long. How long is the hypotenuse, to the nearest tenth of a centimetre?

23. The hypotenuse of a right triangle is 10 m long. How long is the side adjacent to the 21° angle, to the nearest tenth of a metre?

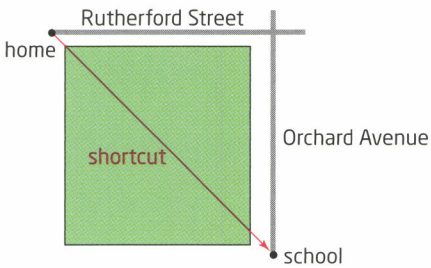
24. A kite string is 35 m long. The angle the string makes with the ground is 50°. To the nearest metre, how far from the person holding the string is a person standing directly under the kite?



25. Find all the angles in $\triangle WXY$, to the nearest degree.



26. To get to school, Enzo can travel 1.2 km east on Rutherford Street and then south on Orchard Avenue to his school. Or, he can take a shortcut through the park, as shown. His shortcut takes him 20 min.

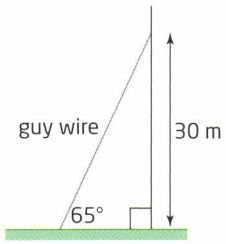


Enzo's walking speed is 6 km/h.

a) What angle does Enzo's shortcut make with Rutherford Street? Describe any assumptions you must make.

b) How much time does Enzo save by taking his shortcut? Explain your answer.

27. In Example 5, Section 7.3, this problem was solved. A radio transmitter is to be supported with a guy wire. The wire is to form a 65° angle with the ground and reach 30 m up the transmitter. The wire can be ordered in whole-number lengths of metres. How much wire should be ordered?



Solve this problem using a more efficient method.

28. How are the sines and cosines of the acute angles of a right triangle related? Plan and carry out an investigation to explore this. Write a brief report of your findings, using words, diagrams, and mathematical notation.

29. a) Is it possible for the sine or cosine of an angle to be greater than 1? Use geometric reasoning to explain your answer.

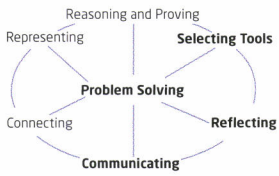
b) Is it possible for the tangent of an angle to be greater than 1? Use geometric reasoning to explain your answer.

30. Chapter Problem A right triangle is formed by the following locations:

- your current location (in the race)
- the capital of the United States
- your next destination

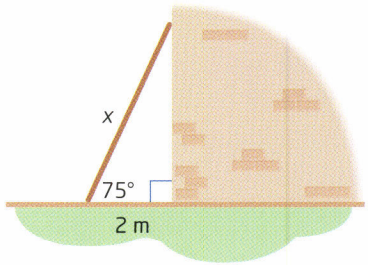
The cosine of the angle at your current location is approximately 0.8. What city is your next destination, and how far do you have to travel to get there?

Achievement Check



31. When it is leaning against a wall, the foot of a ladder is 2 m from the base of the wall. The angle between the ladder and the ground is 75°.

a) How high up the wall does the ladder reach, to the nearest centimetre?

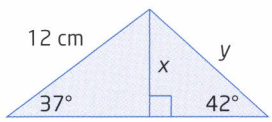


b) How long is the ladder, to the nearest centimetre?

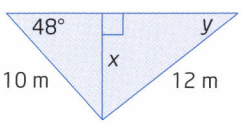
c) If the ladder slips down the wall so that it makes an angle of 55° with the ground, does the end on the ground slip more than the end against the wall? Explain.

Extend

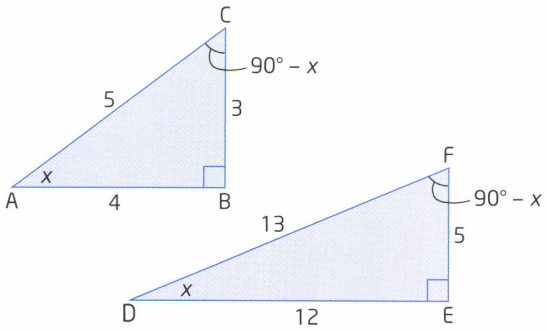
32. Find the length of x, then the length of y, to the nearest tenth of a centimetre.



33. Find the length of x, to the nearest tenth of a metre, then the measure of y, to the nearest degree.



34. a) Use △ABC and △DEF. Copy and complete the table. Leave all ratios in fraction form.



Triangle	△ABC	△DEF
tan x		
sin x		
cos x		
tan (90° – x)		
sin (90° – x)		
cos (90° – x)		

b) How is tan x related to tan $(90° - x)$ in these two triangles?

c) How is sin x related to cos $(90° - x)$ in these two triangles?

d) How is cos x related to sin $(90° - x)$ in these two triangles?

e) Explain the relationships in parts b), c), and d).

35. Use geometric reasoning to show that $\sin\theta = \cos(90° - \theta)$ in all right triangles, if θ is one of the acute angles.

36. Use algebraic reasoning to show that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

37. a) Select three different values for an angle x between 0° and 90°. For each value of x, evaluate the expression $(\sin x)^2 + (\cos x)^2$.

b) Use your results from part a) to write a conjecture.

c) Verify your conjecture from part b) using geometric reasoning.

38. Pilots use a "wind triangle" to determine which way to aim the aircraft to overcome the effects of wind. For example, Seymour has an airplane that cruises at 200 km/h in still air. A stiff wind of 28 km/h is blowing from the west. Seymour wants to fly from A directly north to B.

a) Find the angle at which Seymour must aim the airplane.

b) How fast will he be flying relative to the ground?

39. Math Contest At the senior prom, four couples are seated randomly around a circular table. What is the probability that Dan sits beside his date Ranjit?

A $\frac{1}{4}$

B $\frac{1}{8}$

C $\frac{1}{2}$

D $\frac{2}{7}$

E $\frac{3}{8}$

40. Math Contest An equilateral triangle and a hexagon have equal perimeters. The area of the triangle is 2 m^2. What is the area of the hexagon?

41. Math Contest The lengths of the sides of a triangle are 20 cm, 21 cm, and 29 cm. The shortest distance from the longest side to the opposite vertex is

A $\frac{400}{9}$ cm

B $\frac{410}{29}$ cm

C $\frac{420}{29}$ cm

D $\frac{580}{21}$ cm

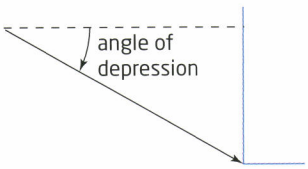
E $\frac{609}{20}$ cm

7.5 Solve Problems Involving Right Triangles

The primary trigonometric ratios are applied in many areas of study, including architecture, engineering, astronomy, medicine, and criminal detection. For example, have you ever seen a television show or movie in which forensic evidence is used to recreate the events of an accident or crime? How can scientists use a few tiny clues, such as bloodstains, to piece together what actually happened? How could trigonometry be useful?

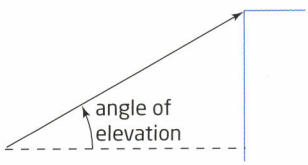
angle of depression

- angle measured below the horizontal
- also called the **angle of declination**



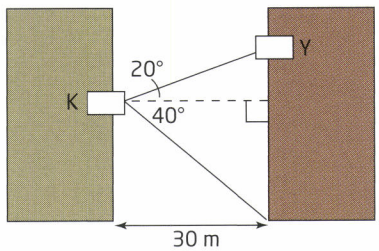
angle of elevation

- angle measured above the horizontal
- also called the **angle of inclination**



Example 1 Angles of Depression and Elevation

Kim and Yuri live in apartment buildings that are 30 m apart, as shown. The **angle of depression** from Kim's balcony to where Yuri's building meets the ground is 40°. The **angle of elevation** from Kim's balcony to Yuri's balcony is 20°.

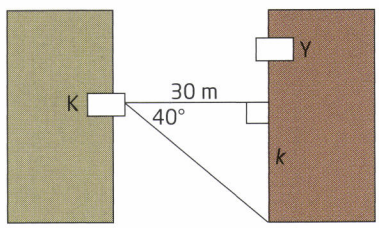


a) How high is Kim's balcony above the ground, to the nearest metre?

b) How high is Yuri's balcony above the ground, to the nearest metre?

Solution

a) Simplify the problem by focusing on the lower right triangle.



$$\tan 40° = \frac{\text{opposite}}{\text{adjacent}}$$

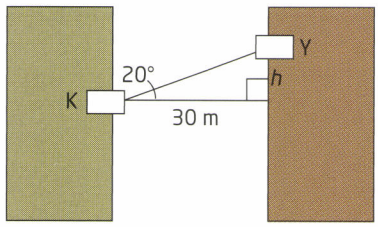
$$\tan 40° = \frac{k}{30}$$

$30 \tan 40° = k$ Multiply both sides by 30.

$25.17 \doteq k$

Kim's balcony is about 25 m above the ground.

b) Focus on the upper right triangle.



$$\tan 20° = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 20° = \frac{h}{30}$$

$30 \tan 20° = h$ Multiply both sides by 30.

$10.92 \doteq h$

Yuri's balcony is about 11 m higher than Kim's. Add this to the height of Kim's balcony to find the height of Yuri's balcony, y.

$y = h + k$

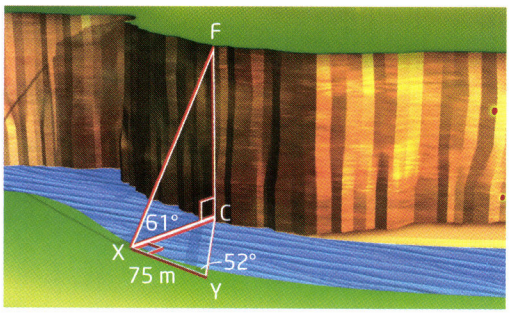
$= 25 + 11$

$= 36$

Yuri's balcony is about 36 m above the ground.

Example 2 Solve a Three-Dimensional Problem

A theodolite is an instrument used by a surveyor to measure horizontal and vertical angles. Measurements are taken in order to find the height of a cliff on the other side of a river, as shown.



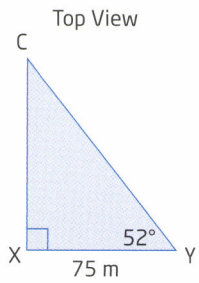
Find the height of the cliff, to the nearest metre.

Literacy Connections

In this situation, using a single letter to identify a side can lead to confusion. For example, side c could refer to the hypotenuse in $\triangle CXF$ or the 75-m side of $\triangle CYX$. Use endpoints to distinguish line segments, FX and XY in this case, to avoid confusion.

Solution

Use $\triangle CYX$ to find the width of the river, CX. Then, use $\triangle CXF$ to find the height of the cliff, CF. Focus on $\triangle CYX$ first.



Apply the tangent ratio to find CX.

$$\tan \angle Y = \frac{CX}{XY}$$

$$\tan 52° = \frac{CX}{75}$$

$75 \tan 52° = CX$ Multiply both sides by 75.

$95.996 \doteq CX$

CX is shared by both triangles. Use it to find the height of the cliff.

Focus on $\triangle CXF$.

Apply the tangent ratio to find CF.

$$\tan \angle X = \frac{CF}{CX}$$

$$\tan 61° = \frac{CF}{96}$$

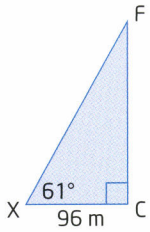
$$96 \tan 61° = CF$$

Multiply both sides by 96.

$$173.2 \doteq CF$$

The cliff is about 173 m high.

Side View



Key Concepts

- Angles of elevation and depression are measured above and below a horizontal line, respectively.
- The primary trigonometric ratios can be applied to solve two-dimensional and three-dimensional problems involving right triangles.
- Some complex problems involve working with more than one right triangle.

Communicate Your Understanding

C1 **a)** What is an angle of elevation?

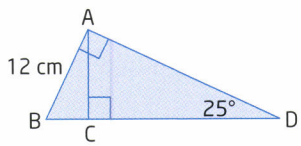
b) How is it measured?

C2 **a)** What is an angle of depression?

b) How is it measured?

C3 **a)** What triangles can you identify in the diagram?

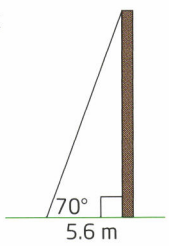
b) Describe the steps you would use to find the side length CD.



Practise

For help with questions 1 and 2, see Example 1.

1. A telephone pole is secured at its top with a guy wire, as shown. The guy wire makes an angle of 70° with the ground and is secured 5.6 m from the bottom of the pole. Find the height of the telephone pole.



2. a) Find the length of the guy wire in question 1.

b) Use a different method to find the length of the guy wire in question 1.

For help with questions 3 to 7, see Example 2.

3. Refer to question 1. A second guy wire is to be added to support the pole. It is to be secured on the ground twice as far from the pole as the first wire, on the same side of the pole and attached to the top of the pole.

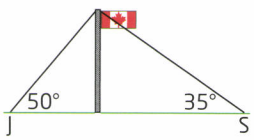
a) Draw a diagram illustrating the telephone pole and both guy wires.

b) Find the length of the second wire and the angle it will make with the ground.

c) Find the angle formed between the two wires at the top of the pole.

4. Refer to question C3. Solve for side length CD. Record your answer to the nearest centimetre.

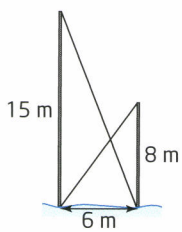
5. Jack and Sangita are facing each other on opposite sides of a 10-m flagpole. From Jack's point of view, the top of the flagpole is at an angle of elevation of 50°. From Sangita's it is 35°.



How far apart are Jack and Sangita?

6. Alexa and Emma are looking up at their house from the backyard. From Alexa's point of view, the top of the house is at an angle of elevation of 40°. From Emma's point of view, directly closer to the house, it is 60°. The house is 15 m high. How far apart are the two girls?

7. At the bottom of a ski lift, there are two vertical poles: one 15 m tall and the other 8 m tall. The ground between the poles is level, and the bases of the poles are 6 m apart. The poles are connected by two straight wires.



a) What angle does each wire make with the ground?

b) What is the length of each wire?

8. Refer to question 7.

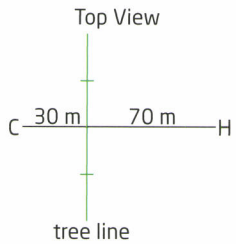
a) Use a different method to answer part b).

b) At what height above the ground do the wires intersect, to the nearest tenth of a metre?

Connect and Apply

Use this information to answer questions 9 and 10.

Cheryl is golfing. She is 100 m from the hole, which is her target. Blocking her direct path is a line of trees, the midpoint of which is 30 m from her current position. She has two choices:

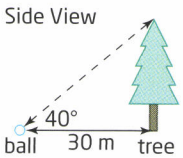


- Option 1: Aim directly for the hole, over the trees.
- Option 2: Go around the trees in two shots.

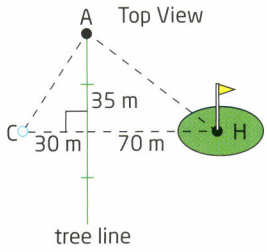
Cheryl's average distances using various clubs are shown in the table.

Club	Distance (m)
Lob wedge	25
Sand wedge	50
Pitching wedge	90
9-iron	100

9. Cheryl considers Option 1, to aim over the trees. The closest tree, which is in line with the hole, is about 30 m away, as shown. She estimates that the angle of inclination from her ball to the top of the tree is about 40°. She judges that the maximum height she can hit from this position is 20 m. Should she take this shot? Explain why or why not.



10. Cheryl considers Option 2, to go around the tree in two shots. In shot one, she will shoot to the end of the line of trees to position A. In shot two, she will aim for the hole.



The tree line is 70 m long, and the line joining Cheryl's ball and the hole, CH, passes through the middle of the tree line, at a right angle.

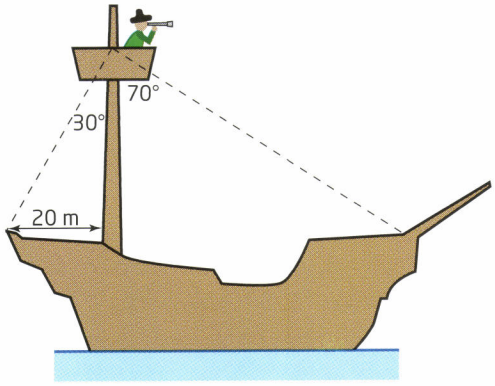
a) At what angle from CH should Cheryl make her first shot, in order to land near A?

b) Which club should she choose for this shot? Explain.

c) Assume that Cheryl succeeds with her first shot and her golf ball lands at A. At what angle from the tree line must she aim for her second shot?

d) Which club should she choose for this shot? Explain.

11. Captain Jack is sitting in the crow's-nest of his ship, as shown.

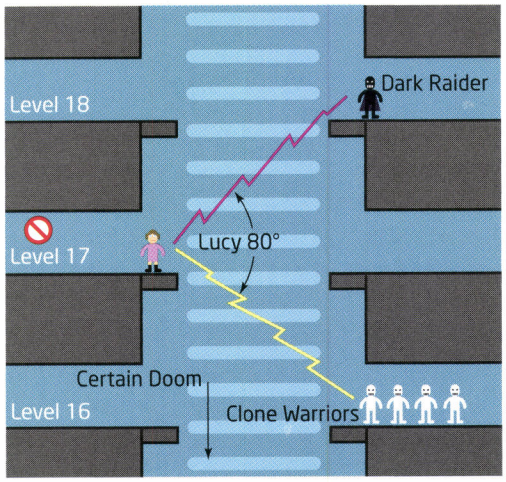


a) How high above the deck is Captain Jack?

b) What is the length of Captain Jack's ship?

c) How long is each wire holding up the crow's-nest?

12. Lucy Starstrider is trapped on Level 17 of a space station. The evil Dark Raider and his Clone Warriors, who are on Levels 18 and 16 across a deep trench, face her. Lucy's retreat path behind her has been blocked. Her only chance for escape is to try to jump directly across the trench. If she does not make it, she will fall to certain doom.



Lucy estimates that the lines of sight to her enemies are 80° apart, as shown. She also recalls, from the technical plans of the space station, that consecutive levels are 10 m apart, vertically. Using the Source of Power, Lucy can leap a horizontal distance of 12 m. Time is quickly running out! Will Lucy Starstrider escape the clutches of the evil ones, or will she perish? Justify your answer.

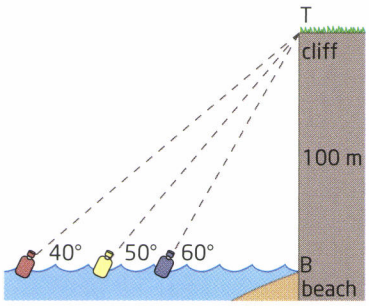
13. A scuba diver swam north at 1.5 m/s, across a current running from east to west at 2.0 m/s. She swam for 3 min and then surfaced.

a) Draw a diagram showing where the dive boat will pick her up relative to where she dove.

b) How far did she travel?

14. The observation deck at Peggy's Cove lighthouse, in Nova Scotia, is about 20 m above sea level. From the observation deck, the angle of depression of a boat on the water is 6°. How far is the boat from the lighthouse, to the nearest metre?

15. Theresa and Branko are competing in a series of outdoor challenges that will eventually lead them to a hidden treasure. Each clue they find helps them find a new clue. Theresa is at the top of a cliff that she knows to be 100 m high, looking down at three anchored floating bottles, as shown.



She reads the clue that she and Branko just found:

> From the top of the cliff, find the bottle whose angle of depression is 50°.

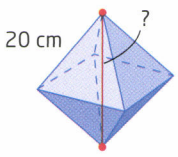
Branko is waiting on the beach below for instructions from Theresa.

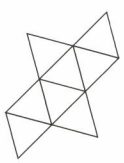
a) What colour bottle should Theresa tell Branko to look for?

b) How far out should she tell him to swim?

16. Refer to Example 1. Yuri's balcony is four floors above Kim's. What floors do they live on? Explain any assumptions you must make.

17. An octahedron is formed by attaching eight congruent equilateral triangles, as shown.

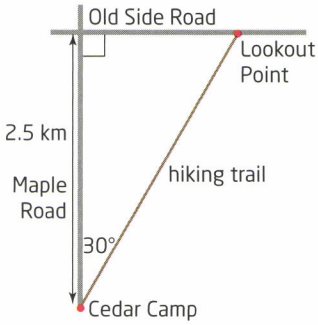




net of octahedron

If the length along one of the edges is 20 cm, find the distance between opposite vertices.

18. Joanne and Sandy are hiking from Cedar Camp to Lookout Point along the hiking trail shown.



Cedar Camp is 2.5 km from Old Side Road along Maple Road, which runs flat. The hiking trail makes an angle of 30° with Maple Road and climbs at an average angle of elevation of 15°.

a) How far apart would Cedar Camp and Lookout Point appear, according to a normal map?

b) What distance do the hikers actually walk? Why are these answers different?

c) What is the difference in elevation between Lookout Point and Cedar Camp?

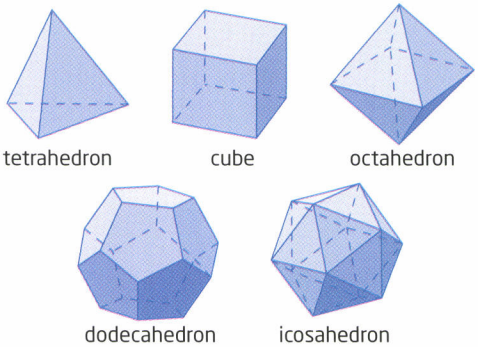
d) What is the average angle of elevation of the section of Old Side Road that is shown?

Making Connections

The octahedron is one of five Platonic solids, polyhedra that are formed by attaching congruent regular polygons.

The Platonic solids are named after Plato, a great Greek mathematician and teacher, who lived from 427 to 347 B.C.E.

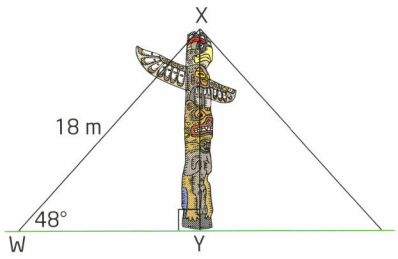
Plato is famous for founding the world's first known university in Athens, Greece, called The Academy.



tetrahedron cube octahedron

dodecahedron icosahedron

19. Ropes are used to pull a totem pole upright. Then, the ropes are anchored in the ground to hold the pole until the hole is filled. One of the ropes holding this totem pole is 18 m long and forms an angle of 48° with the ground.



a) Find the height of the totem pole, to the nearest metre.

b) How far is the anchor point from the base of the totem pole, to the nearest metre?

20. Edmonton's CN Tower is a highrise office building. From a point 35 m from the base of the building and level with the base, the angle of elevation of the top is 72.5°. Find the height of Edmonton's CN Tower, to the nearest metre.

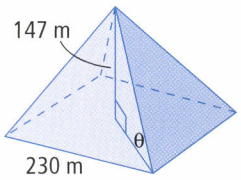
21. A coast guard patrol boat is 14.8 km east of the Brier Island lighthouse. A disabled yacht is 7.5 km south of the lighthouse.

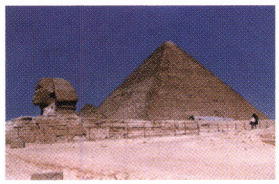
a) How far is the patrol boat from the yacht, to the nearest tenth of a kilometre?

b) At what angle south of due west, to the nearest degree, should the patrol boat travel to reach the yacht?

22. The Capilano Suspension Bridge in North Vancouver is the world's highest footbridge of its kind. The bridge is 140 m long. From the ends of the bridge, the angles of depression of a point on the river under the bridge are 41° and 48°. How high is the bridge above the river, to the nearest metre?

23. The Great Pyramid of Cheops is a square-based pyramid with a height of 147 m and a base length of 230 m. Find the angle, to the nearest degree, that one of the edges of the pyramid makes with the base.





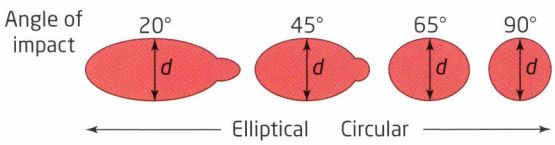
24. Chapter Problem The city you are now in is next to a large river. Take a boat downstream to another city located at the river's end. From there you must fly to a town known as the Gateway to the Yukon. This town, your current location in the race, and Canada's capital form a right triangle. The tangent of the angle at the city you have just sailed to is 1.75. What is your final destination?

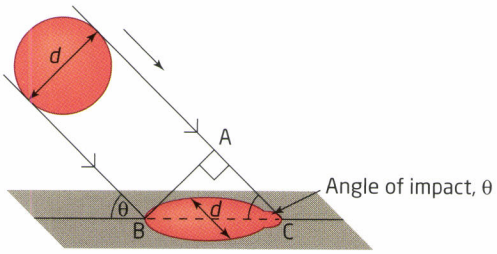
Extend

Use this information to answer questions 25 and 26.

Forensic scientists can recreate an accident or crime by examining bloodstains. A blood droplet starts out in the shape of a sphere. When it falls straight down to the floor, it usually forms a circle with the same diameter as the sphere.

However, when blood hits the floor at an angle, due to the force of a blow, the circle becomes elongated into a shape called an ellipse. The ellipse's width is the same as the sphere's diameter, but because of the force in the direction of motion, its length is greater than the sphere's diameter.

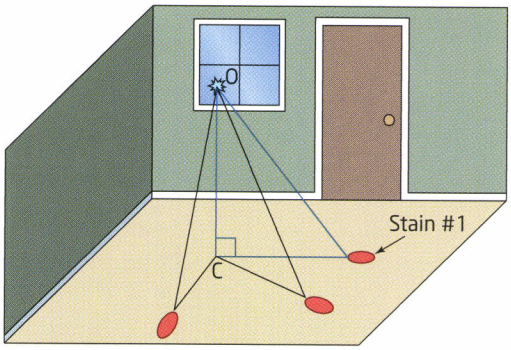




The angle of impact, θ, is the acute angle formed between the path of the blood drop and the floor.

25. Use geometric reasoning to show that the angle of impact can be found using the relationship $\sin\theta = \frac{d}{BC}$.

26. Three bloodstains from a victim are shown. The point of convergence, C, has been found by extrapolating the directions of these stains along the floor. The origin of the blow, O, is some height above C.



Forensic analysis of Stain #1 provides the following data.

Length of bloodstain:	4.2 cm
Width of bloodstain:	2.6 cm
Distance from point of convergence	2.1 m

Determine the height at which the blow struck the victim.

27. A sign shows that a hill has a grade of 9%. What angle does the hill make with the horizontal, to the nearest tenth of a degree?

28. An airplane is cruising at an altitude of 10 000 m. It is flying in a straight line away from Chandra, who is standing on the ground. If she sees the angle of elevation of the airplane change from 70° to 33° in 1 min, what is its cruising speed, to the nearest kilometre per hour?

29. A special type of aircraft is designed to fly at the very low height of 20 m. To measure such a small altitude, two spotlights are mounted on the aircraft:

- one on the nose, pointing straight down
- another mounted on the tail of the aircraft, 10 m away

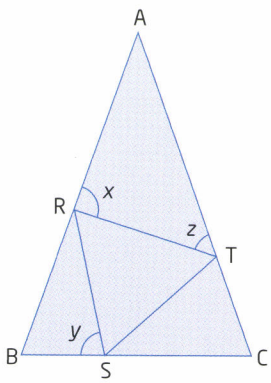
Find the angle at which the second light needs to be set, with respect to the body of the aircraft, so that the beams will meet 20 m below the aircraft.

30. The angle of elevation to a building is 30°. From a point 20 m directly toward the building, the angle of elevation changes to 40°. Find the height of the building. Include a diagram with your solution.

31. Math Contest In a family of four children, what is the probability that there are at least two girls if the eldest child is a girl?

A $\frac{15}{16}$ **B** $\frac{7}{16}$ **C** $\frac{3}{4}$ **D** $\frac{7}{8}$ **E** $\frac{3}{8}$

32. In the diagram, $\triangle ABC$ is isosceles, with AB = AC, and $\triangle RST$ is equilateral. Express $\angle x$ in terms of $\angle y$ and $\angle z$.



Chapter 7 Review

7.1 Investigate Properties of Similar Triangles, pages 330–335

1. a) Explain the difference between similar and congruent figures.

b) Draw two triangles that are congruent.

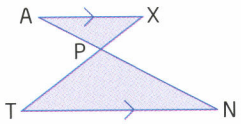
c) Draw two rectangles that are similar.

d) Draw two pentagons that are similar.

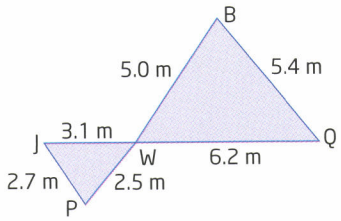
e) Draw two non-similar pentagons whose corresponding sides are in the same ratio.

2. Are all circles similar? Justify your reasoning.

3. Name the two similar triangles and explain why they are similar.



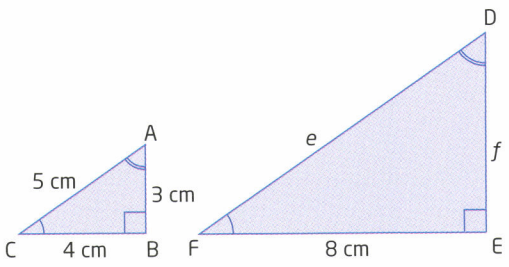
4. Name the two similar triangles and explain why they are similar.



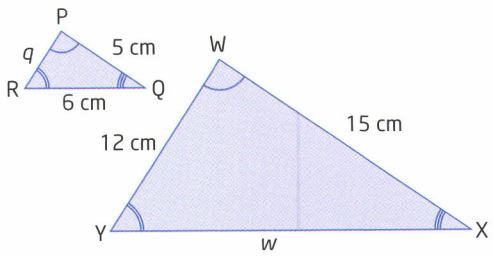
7.2 Use Similar Triangles to Solve Problems, pages 342–351

5. The pairs of triangles are similar. Find the unknown side lengths.

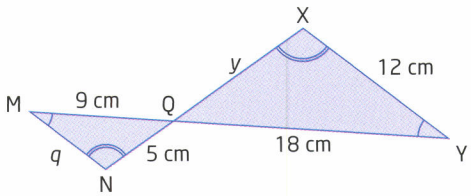
a)



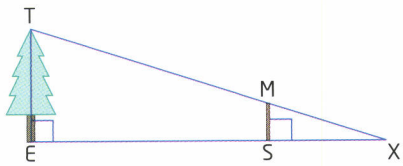
b)



c)



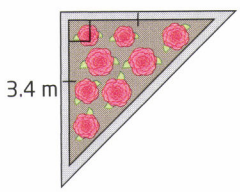
6. The tips of the shadows of a tree and of a metre stick meet at the point X.



The following measurements are taken:
XS = 3.5 m
ES = 6.5 m

Use this information to find the height of the tree, to the nearest tenth of a metre.

7. Janke's garden is in the shape of a right isosceles triangle with base 3.4 m. If she enlarges her garden to a similar shape whose base is doubled, what will the area of her new garden be?

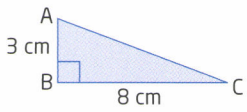


8. Sid wants to find the height of a tree without having to climb it, but it is a cloudy day, so he cannot use shadows. He takes a mirror from his pocket and places it on the ground 7.2 m from the base of the tree. He backs up until he can see the top of the tree in the mirror, a distance of 1.2 m from the mirror. If Sid's eyes are 1.5 m above the ground, what is the height of the tree?

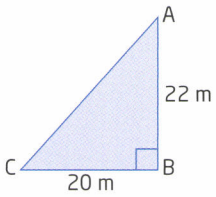
7.3 The Tangent Ratio, pages 352–365

9. Find the measure of $\angle A$, to the nearest degree.

a)

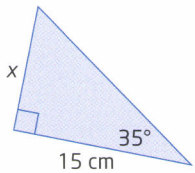


b)

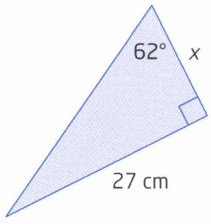


10. Find x, to the nearest tenth of a centimetre.

a)



b)



11. A jet climbs at a steady rate at an angle of inclination of 0.5° during take-off. What will its height be after a 2.2-km initial ascent at this angle?

12. Cory is building a ramp for his school theatre production. It must climb a height of 60 cm and have a slope angle of 10°.

a) Draw a diagram and label the given information.

b) What distance will the ramp run along the floor?

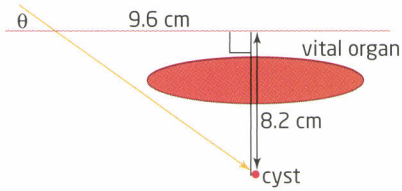
c) What will the distance along the surface of the ramp be?

13. a) What are the following values?

- tan 0°
- tan 45°
- tan 90°

b) Use words and diagrams to explain these values.

14. To avoid damaging a vital organ, a surgeon will fire a laser at an angle to the patient's skin, to reach a cyst (an abnormal growth). The cyst is 8.2 cm directly below the skin, and the laser is positioned at a distance of 9.6 cm away in order to miss the vital organ, as shown.

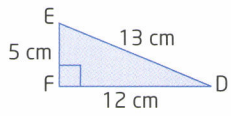


At what angle, θ, should the surgeon position the laser with respect to the skin's surface?

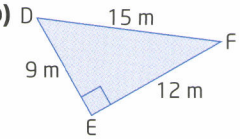
7.4 The Sine and Cosine Ratios, pages 366–377

15. Find the measure of both acute angles in each triangle, to the nearest degree.

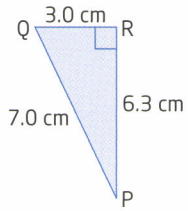
a)



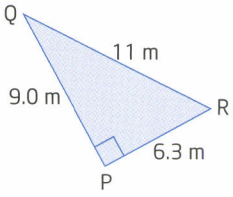
b)



c)

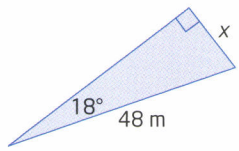


d)

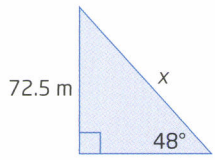


16. Find x, to the nearest tenth of a unit.

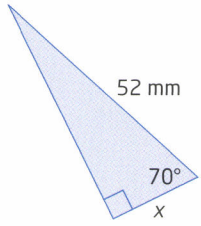
a)



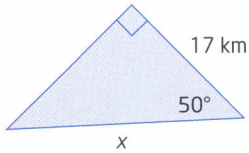
b)



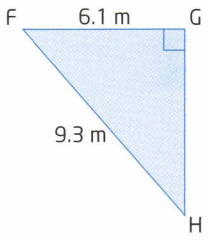
c)



d)



17. Solve $\triangle FGH$.

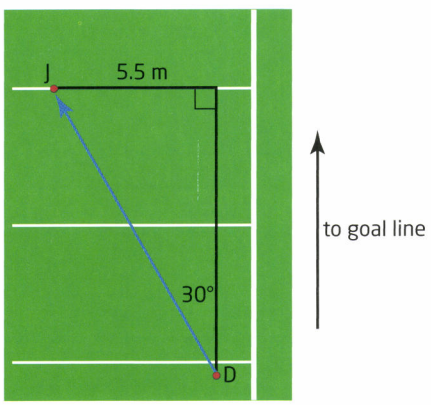


18. In $\triangle TUV$,
$UV = 7.4$ km
$\angle U = 90°$
$\angle T = 38°$

a) Draw the triangle and label the given information.

b) Solve $\triangle TUV$.

19. During a football game, Danny, the quarterback, has the football and is facing the other team's goal line. His receiver, Javier, is about 5.5 m to Danny's left, at an angle of 30°, as shown.



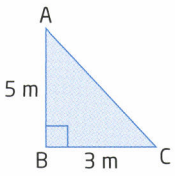
How far should Danny throw the ball to his receiver, to the nearest metre?

20. Rachel is heading due south in her sailboat toward a port. After travelling 12 km, she reaches shore 4 km west of her intended destination, due to the water's current. By what angle did the current push Rachel off course? Include a diagram in your solution and describe any assumptions you must make.

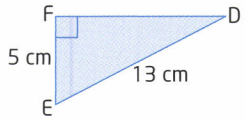
7.5 Solve Problems Involving Right Triangles, pages 378–385

21. Solve each triangle. Round each side length to the nearest tenth of a unit and each angle to the nearest degree.

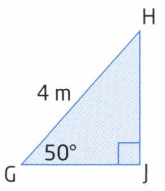
a)



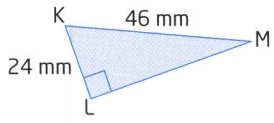
b)



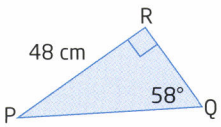
c)



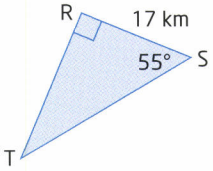
d)



e)

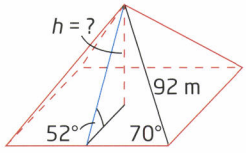


f)



22. The maximum angle of climb for a certain light aircraft is 9°. A line of electric wires 20 m above the ground is located 120 m from the end of the runway. Will the aircraft clear the wires after take-off?

23. A stairway runs up the edge of the pyramid. From bottom to top the stairway is 92 m long.

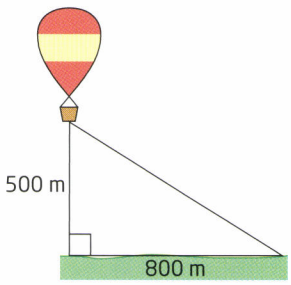


The stairway makes an angle of 70° to the base edge, as shown. A line from the middle of one of the base edges to the top of the pyramid makes an angle of elevation of 52° with respect to the flat ground. Find the height of the pyramid.

24. Kathe is 4 m from the base of a long wooden fence, under which her baseball has just rolled. Kathe estimates that the angle of elevation from where she is to the top of the fence is about 30°. Kathe thinks she can climb over a fence that is a maximum of 2 m high. Can she climb over the fence, or does she have to go around? Justify your reasoning.

25. When a road has a 10% gradient, it means that the road rises 10 m for every 100 m of horizontal distance travelled. What is the angle of inclination of the road, to the nearest degree?

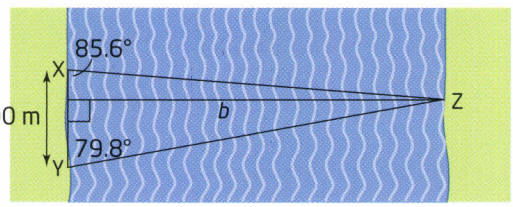
26. If you were in a hot air balloon 500 m above the ground, at what angle of depression would you look at a point on the ground 800 m horizontally from the balloon?



27. A flagpole casts a shadow 28 m long when the Sun's rays make an angle of 25° with the ground. How tall is the flagpole, to the nearest metre?

28. The world's longest escalator is in the subway system in St. Petersburg, Russia. The escalator is 330.7 m long and rises a vertical distance of 59.7 m. What is the angle of elevation of the top of the escalator when viewed from the bottom, to the nearest degree?

29. The world's longest covered bridge crosses the Saint John River in Hartland, New Brunswick. From two points, X and Y, 100 m apart on the same side of the river, the lines of sight to the far end of the bridge, Z, make angles of 85.6° and 79.8° with the river bank, as shown. What is the length of the bridge, b, to the nearest 10 m?



Chapter Problem Wrap-Up

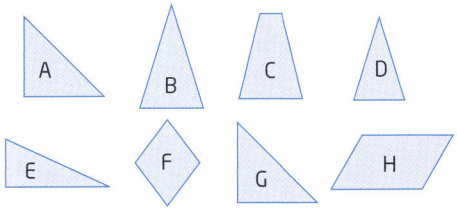
Look back at your answers to question 22 in Section 7.2, question 30 in Section 7.4, and question 24 in Section 7.5. Describe your journey from the start to the finish line, including

- the names of all your destinations
- directions
- distances

Include a diagram to help you illustrate your description.

Chapter 7 Practice Test

1. Identify a pair of similar figures and a pair of congruent figures. Explain your choices.



2. Are all rectangles similar? Justify your answer.

3. Are all squares similar? Justify your answer.

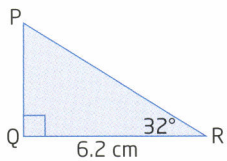
4. Evaluate the following, to four decimal places.

a) tan 29°
b) cos 78°
c) sin 90°
d) sin 45°
e) cos 45°
f) tan 85°

5. Solve for each angle, to the nearest degree.

a) $\sin \theta = 0.8872$
b) $\tan \theta = 1.443$
c) $\cos C = 0.3561$
d) $\cos \theta = 0.2147$
e) $\tan X = 5.3212$
f) $\sin P = 0.9111$

6. Solve $\triangle PQR$. Express lengths to the nearest tenth of a centimetre and angles to the nearest degree.

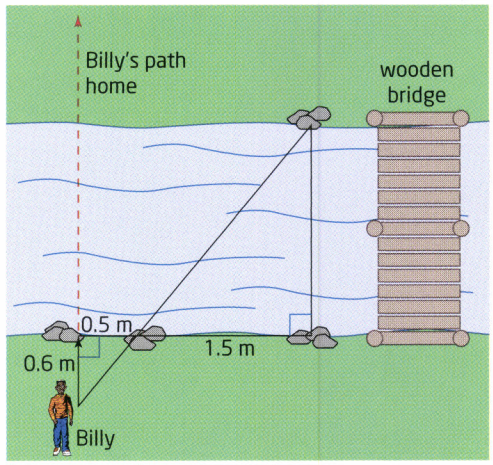


7. In $\triangle ABC$,
$AB = 19$ m
$BC = 27$ m
$\angle B = 90°$

a) Draw this triangle and label the given information.

b) Solve $\triangle ABC$. Express lengths to the nearest metre and angles to the nearest degree.

8. Billy has lost track of time. He will get in trouble if he is late for dinner, but he will get in *more* trouble if he comes home with wet shoes. Racing home, Billy suddenly encounters a creek, as shown. Glancing at some nearby rocks, he estimates the distances indicated.



Billy can long jump about 2 m. Should he try to jump the creek, or take the long way home across the wooden bridge? Justify your reasoning.

9. Salma is at the top of a cliff looking down at Rico's boat. They both have Global Positioning System (GPS) devices and are communicating via cell phones. They determine that Rico's boat is 5.0 km from a point on the shore directly below Salma, and 6.0 km from Salma herself.

a) Draw a diagram to represent this situation and label the given information.

b) Find the angle of depression at which Salma is viewing Rico's boat.

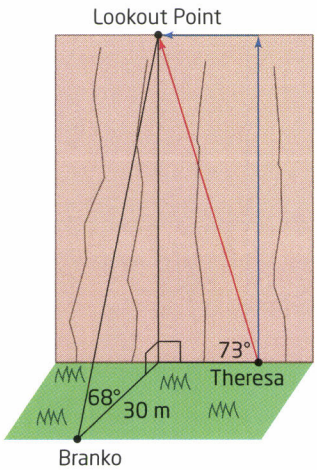
c) Find the height of the cliff.

10. From the top of the CN Tower, the angle of depression to the tip of the tower's shadow is 88°. The shadow is 19.5 m long. How tall is the CN Tower?

11. Theresa and Branko are competing in a series of outdoor challenges that will eventually lead them to a hidden treasure. Each clue they find helps them find a new clue. Theresa is getting ready to climb a steep cliff to find their next clue at the Lookout Point. She has two options:

- Option A: Climb straight up the cliff, and then jog over to Lookout Point.
- Option B: Climb directly to Lookout Point along the diagonal shown.

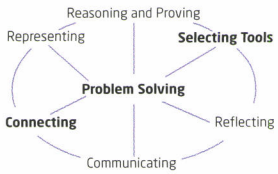
She is awaiting instructions from Branko, who is positioned directly facing Lookout Point at a distance of 30 m from the base of the cliff.



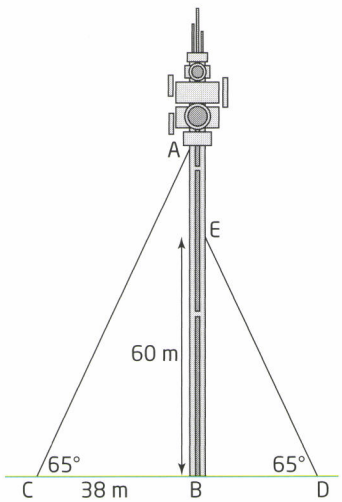
From Branko's point of view, Lookout Point is at an angle of elevation of 68°. He also observes that the diagonal path up the cliff makes a 73° angle with the ground. Branko knows that Theresa can climb at a speed of 1.0 m/s and jog at a speed of 5.0 m/s after a climb. It is a tight race and seconds count. Which option should Branko tell Theresa to take: A or B?

12. A cat, sitting in the top of a tree, spots a dog and a firefighter, both on the flat ground below. From the cat's point of view, the dog is 10 m south, at an angle of depression of 65°, and the firefighter is some distance east of the tree, at an angle of depression of 50°. How far is the firefighter from the dog?

Achievement Check



13. A vertical communications tower is supported by two cables on opposite sides of the tower, as shown in the diagram. One cable is attached to the top of the tower and the other is attached to the tower at a height of 60 m. Both cables are attached to the ground with fasteners.



a) Verify that $\triangle ABC$ is similar to $\triangle EBD$ and list the corresponding sides and angles.

b) What is the height of the tower, to the nearest metre?

c) How long are the supporting cables, to the nearest metre?

d) How far apart are the cable fasteners, to the nearest metre?

Note: This is a simplified diagram. Normally it takes three or four cables to support a tower.